Phase structure and real-time dynamics of the massive Thirring model in 1+1 dimensions using tensor-network methods

Phys. Rev. D 100 (2019) 094504

C.-J. David Lin
National Chiao-Tung University, Taiwan

with Mari Carmen Banuls (MPQ Munich), Krzysztof Cichy (Adam Mickiewicz Univ.),
Hao-Ti Hung (National Taiwan Univ.), Ying-Jer Kao (National Taiwan Univ.),
Yu-Ping Lin (Univ. of Colorado, Boulder), David T.-L. Tan (National Chaio-Tung Univ.)

TNSAA 2019
Taipei
06/12/2019

LGT in the early days

iNSPIRE HEP

Welcome to INSPIRE, the High Energy Physics information system. Please direct questions, comments or concerns to feedback@inspirehep.net.

HEP :: HEPNAMES :: INSTITUTIONS :: CONFERENCES :: JOBS :: EXPERIMENTS :: JOURNALS :: HELP

Information | References (12) | Citations (111) | Files | Plots

{QED} on a Lattice: A Hamiltonian Variational Approach to the Physics of the Weak Coupling Region

S.D. Drell, Helen R. Quinn, Benjamin Svetitsky, Marvin Weinstein (SLAC)

Jun 1978 - 64 pages

Phys.Rev. D19 (1979) 619
DOI: 10.1103/PhysRevD.19.619
SLAC-PUB-2122

Abstract (APS)
We develop and apply a Hamiltonian variational approach to the study of quantum electrodynamics formulated on a spatial lattice in both 2 + 1 and 3 + 1 dimensions. Two lattice versions of QED are considered: a noncompact version which reproduces the physics of continuum QED, and a compact version constructed in correspondence with lattice formulations of non-Abelian theories. Our focus is on photon dynamics with charged particles treated in the static limit. We are especially interested in the nonperturbative aspects of the solutions in the weak-coupling region in order to clarify and establish aspects of confinement. In particular we find, in accord with Polyakov, that the compact QED leads to linear confinement for any nonvanishing coupling, no matter how small, in 2 + 1 dimensions, but that a phase transition to an unconfined phase for sufficiently weak couplings occurs in 3 + 1 dimensions. We identify and describe the causes of confinement.

Keyword(s): INSPIRE: QUANTUM ELECTRODYNAMICS | GAUGE FIELD THEORY: NONABELIAN | APPROXIMATION: LATTICE | FIELD THEORY: THREE-DIMENSIONAL | FIELD THEORY: FOUR-DIMENSIONAL | FIELD THEORY: CRITICAL PHENOMENA | QUARK: CONFINEMENT | FIELD THEORY: NONPERTURBATIVE

Record added 1978-06-01, last modified 2017-02-08

Beginning of MC simulations for LGT

INSPIRE HEP

Welcome to INSPIRE, the High Energy Physics information system. Please direct questions, comments or concerns to feedback@inspirehep.net.

HEP :: HEPNAMES :: INSTITUTIONS :: CONFERENCES :: JOBS :: EXPERIMENTS :: JOURNALS :: HELP

Information | References (31) | Citations (1000) | Files | Plots

Monte Carlo Study of Quantized SU(2) Gauge Theory

M. Creutz (Brookhaven)

1980 - 8 pages

Phys.Rev. D21 (1980) 2308-2315

Reprinted in *Rebbi, C. (ed.): Lattice Gauge Theories and Monte Carlo Simulations*, 258-265

DOI: 10.1103/PhysRevD.21.2308

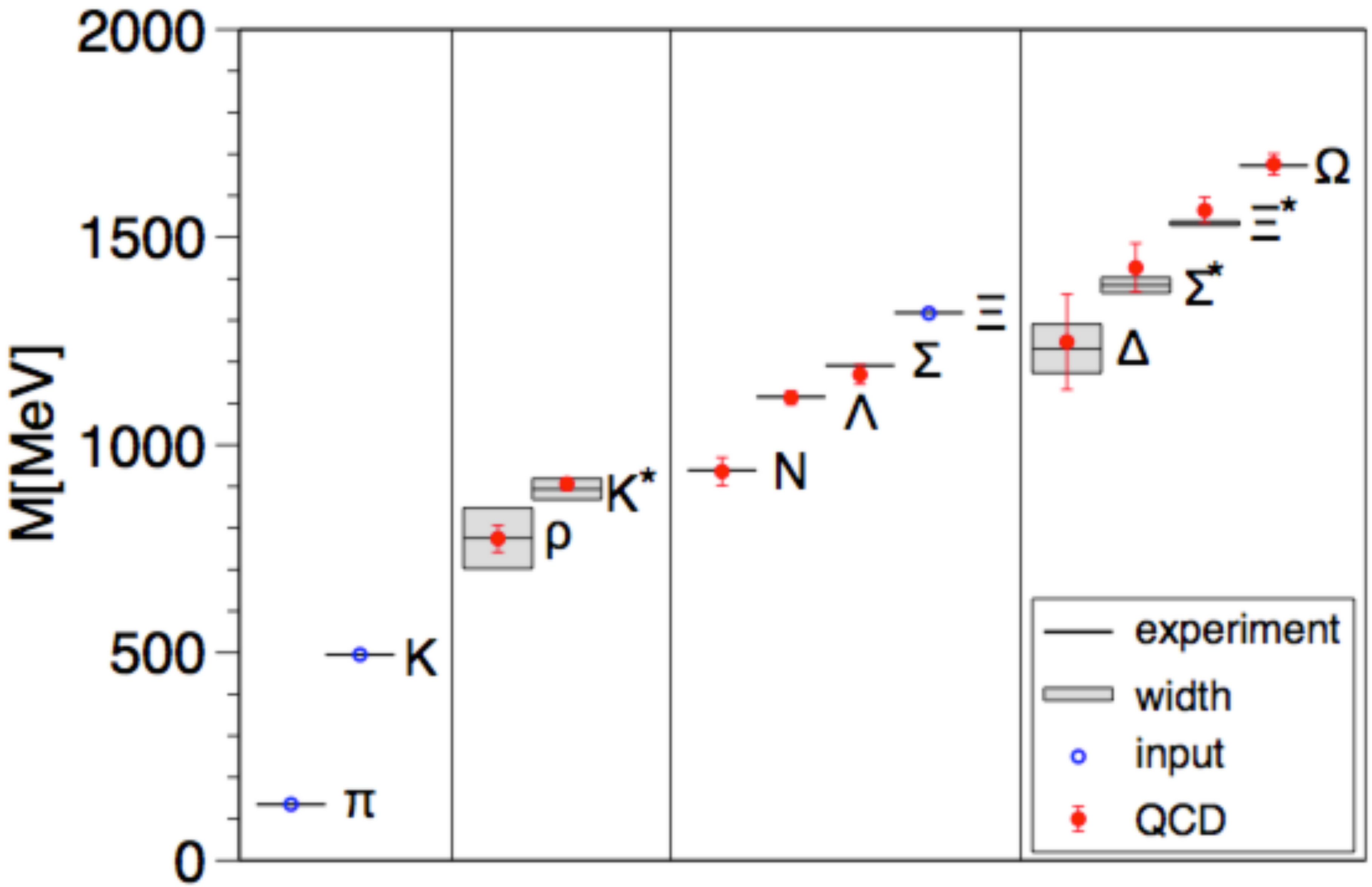
Abstract (APS)

Using Monte Carlo techniques, we evaluate path integrals for pure SU(2) gauge fields. Wilson's regularization procedure on a lattice of up to 104 sites controls ultraviolet divergences. Our renormalization prescription, based on confinement, is to hold fixed the string tension, the coefficient of the asymptotic linear potential between sources in the fundamental representation of the gauge group. Upon reducing the cutoff, we observe a logarithmic decrease of the bare coupling constant in a manner consistent with the perturbative renormalization-group prediction. This supports the coexistence of confinement and asymptotic freedom for quantized non-Abelian gauge fields.

Keyword(s): INSPIRE: GAUGE FIELD THEORY: SU(2) | LATTICE FIELD THEORY: CONTINUUM LIMIT | GAUGE FIELD THEORY: ASYMPTOTIC FREEDOM | GAUGE FIELD THEORY: CONFINEMENT | NUMERICAL CALCULATIONS: MONTE CARLO | LATTICE FIELD THEORY: LAMBDA PARAMETER | GAUGE FIELD THEORY: WILSON LOOP | RENORMALIZATION: REGULARIZATION | COUPLING CONSTANT | RENORMALIZATION GROUP: PERTURBATION THEORY

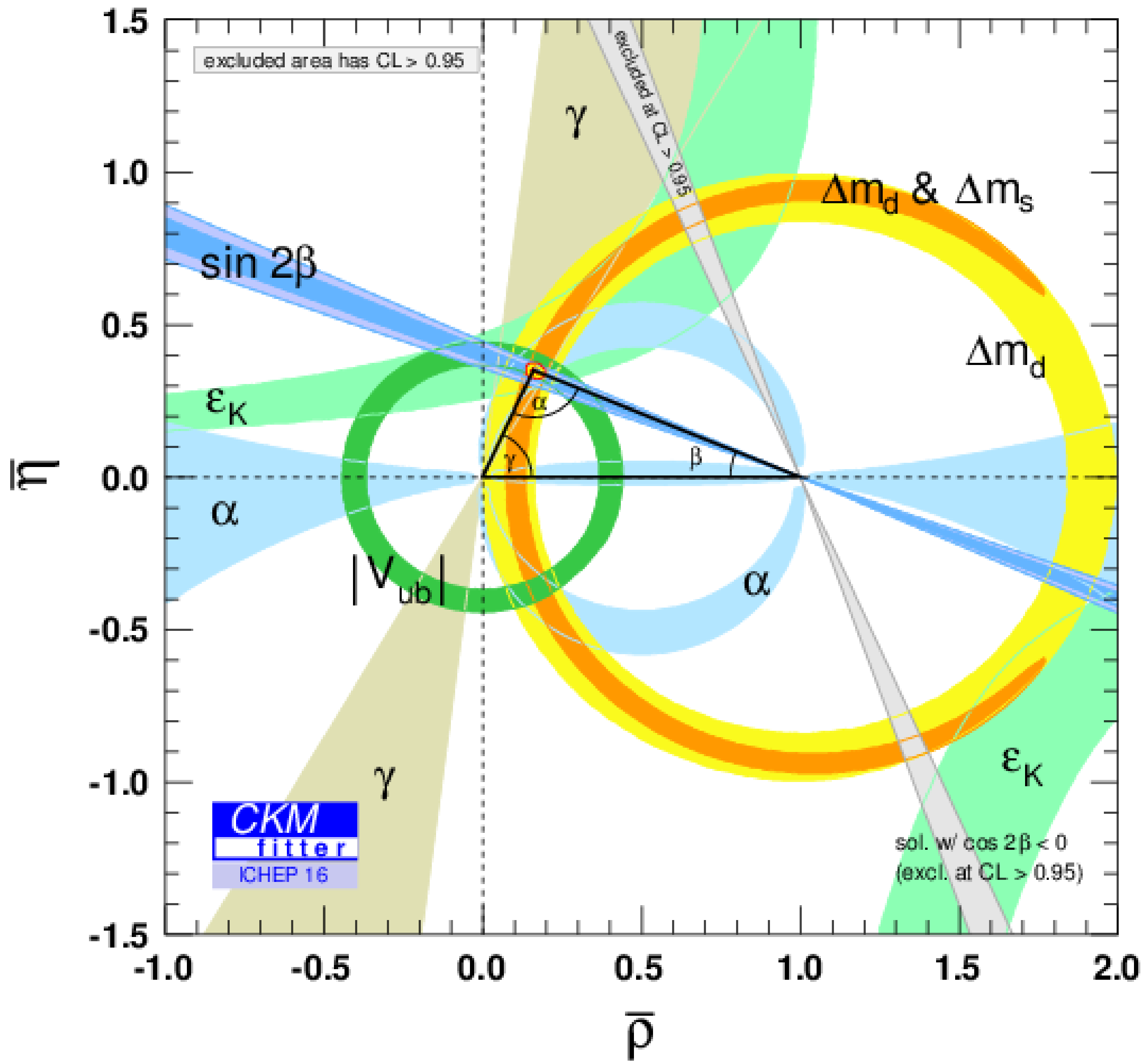
Record added 1980-01-01, last modified 2017-03-28

Success for simple quantities



The BMW collaboration, science 322 (2008)

Success for less simple quantities



G.A. Cowan (LHCb collaboration), arXiv:1708.08628.

Motivation for HEP

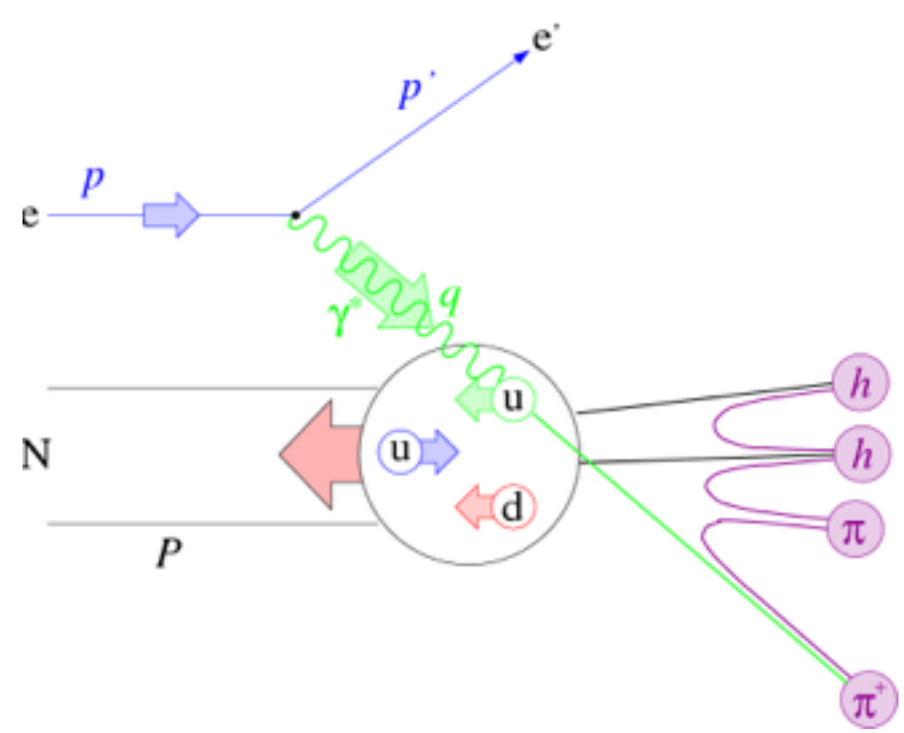
Things that are challenging for Euclidean MC simulations

⋮

See talks by Kuhn and Nakamura

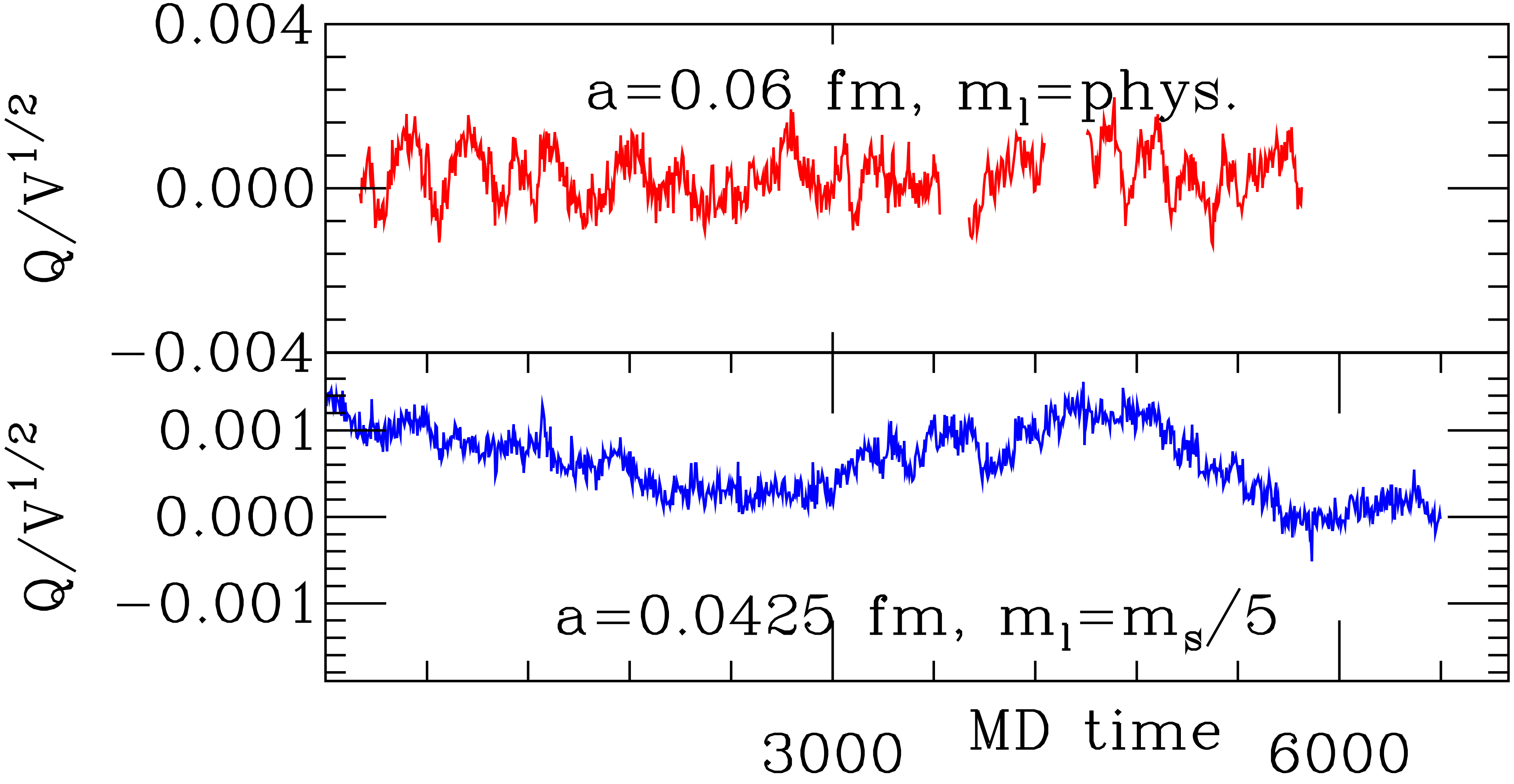
⋮

★ Further examples: light-cone physics, inelastic scattering,…



Motivation for HEP

Topology freezing



Bazavov *et al*., Phys. Rev. D 98 (2018) 074512

Feasibility (toy-model) studies for HEP

The 1+1 dimensional Thirring model and its duality to the sine-Gordon model

$$S_{\mathrm{Th}}\left[\psi,\bar{\psi}\right] = \int d^2x \left[\bar{\psi} i\gamma^\mu \partial_\mu \psi - m_0 \bar{\psi}\psi - \frac{g}{2}\left(\bar{\psi}\gamma_\mu\psi\right)^2\right]$$

strong-weak duality $g \leftrightarrow \kappa$

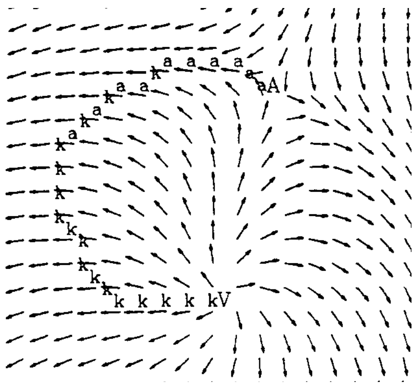
$$S_{\mathrm{SG}}\left[\phi\right] = \int d^2x \left[\frac{1}{2}\partial_\mu \phi(x)\partial^\mu \phi(x) + \frac{\alpha_0}{\kappa^2}\cos\left(\kappa\phi(x)\right)\right]$$

$$\xrightarrow{\phi\to\phi/\kappa,\ \text{and}\ \kappa^2=t} \frac{1}{t}\int d^2x \left[\frac{1}{2}\partial_\mu \phi(x)\partial^\mu \phi(x) + \alpha_0 \cos\left(\phi(x)\right)\right]$$

Works in the zero-charge sector

Dualities and phase structure

Thirring	sine-Gordon	XY
g	$\frac{4\pi^2}{t} - \pi$	$\frac{T}{K} - \pi$



Picture from: K. Huang and J. Polonyi, 1991

★ The K-T phase transition at $T \sim K\pi/2$ in the XY model.

$g \sim -\pi/2$, Coleman's instability point

★ The phase boundary at $t \sim 8\pi$ in the sine-Gordon theory.

The cosine term becomes relevant or irrelevant.

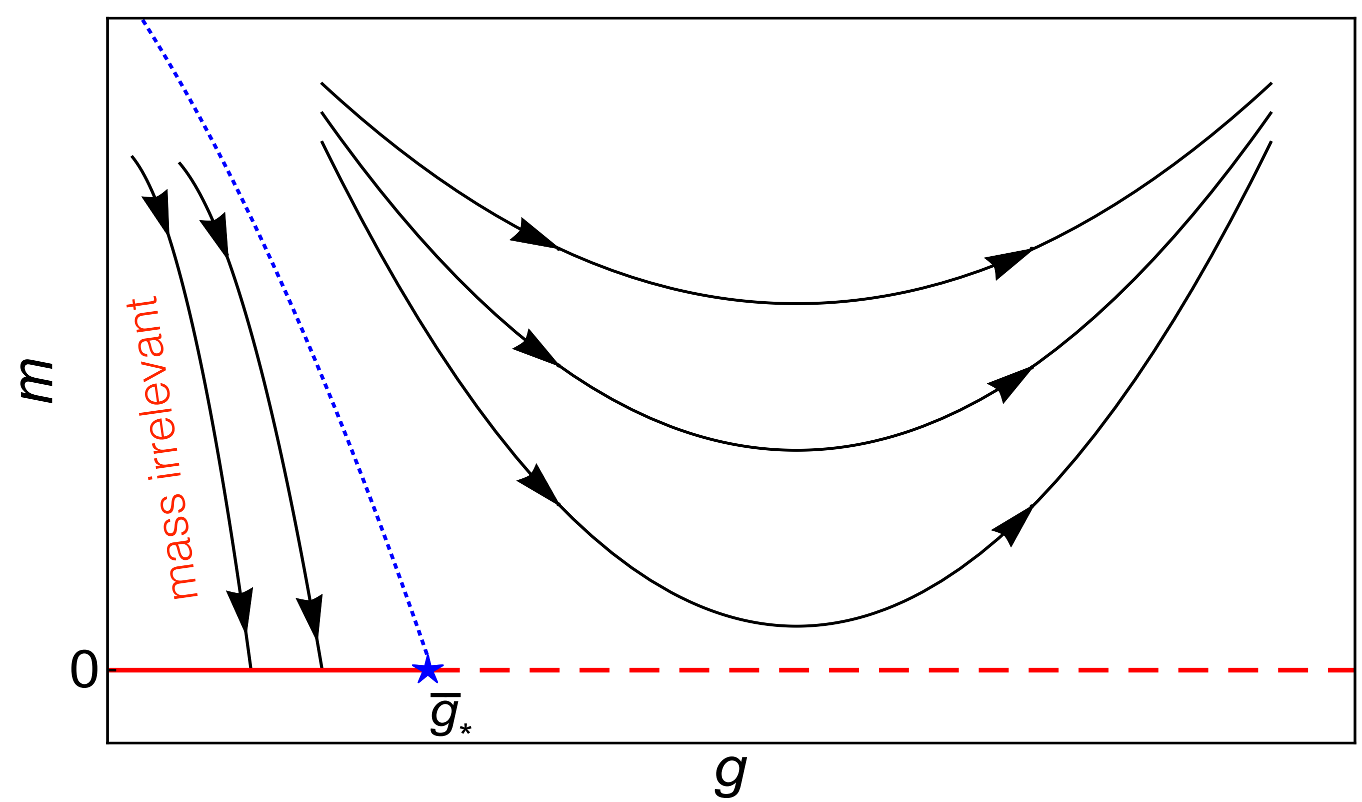
Thirring	sine-Gordon
$\bar{\psi}\gamma_\mu\psi$	$\frac{1}{2\pi}\epsilon_{\mu\nu}\partial_\nu\phi$
$\bar{\psi}\psi$	$\frac{\Lambda}{\pi}cos\phi$

RG flows of the Thirring model

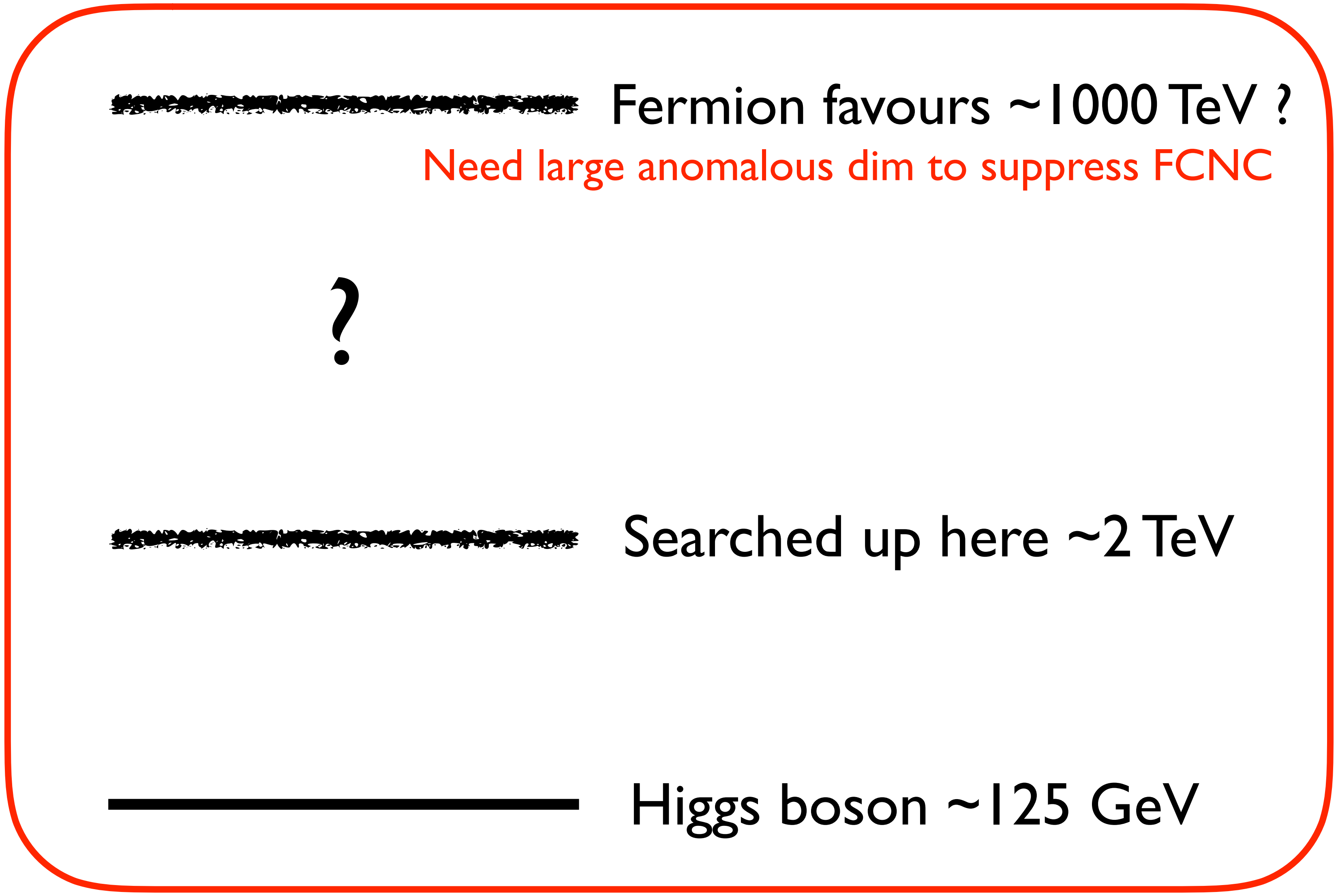
$$\beta_g \equiv \mu \frac{dg}{d\mu} = -64\pi \frac{m^2}{\Lambda^2},$$

$$\beta_m \equiv \mu \frac{dm}{d\mu} = \frac{-2(g+\frac{\pi}{2})}{g+\pi} m - \frac{256\pi^3}{(g+\pi)^2\Lambda^2} m^3$$

★ Massless Thirring model is a conformal field theory



Beyond the SM, composite Higgs?



The Higgs boson is light

The “conformal windows”

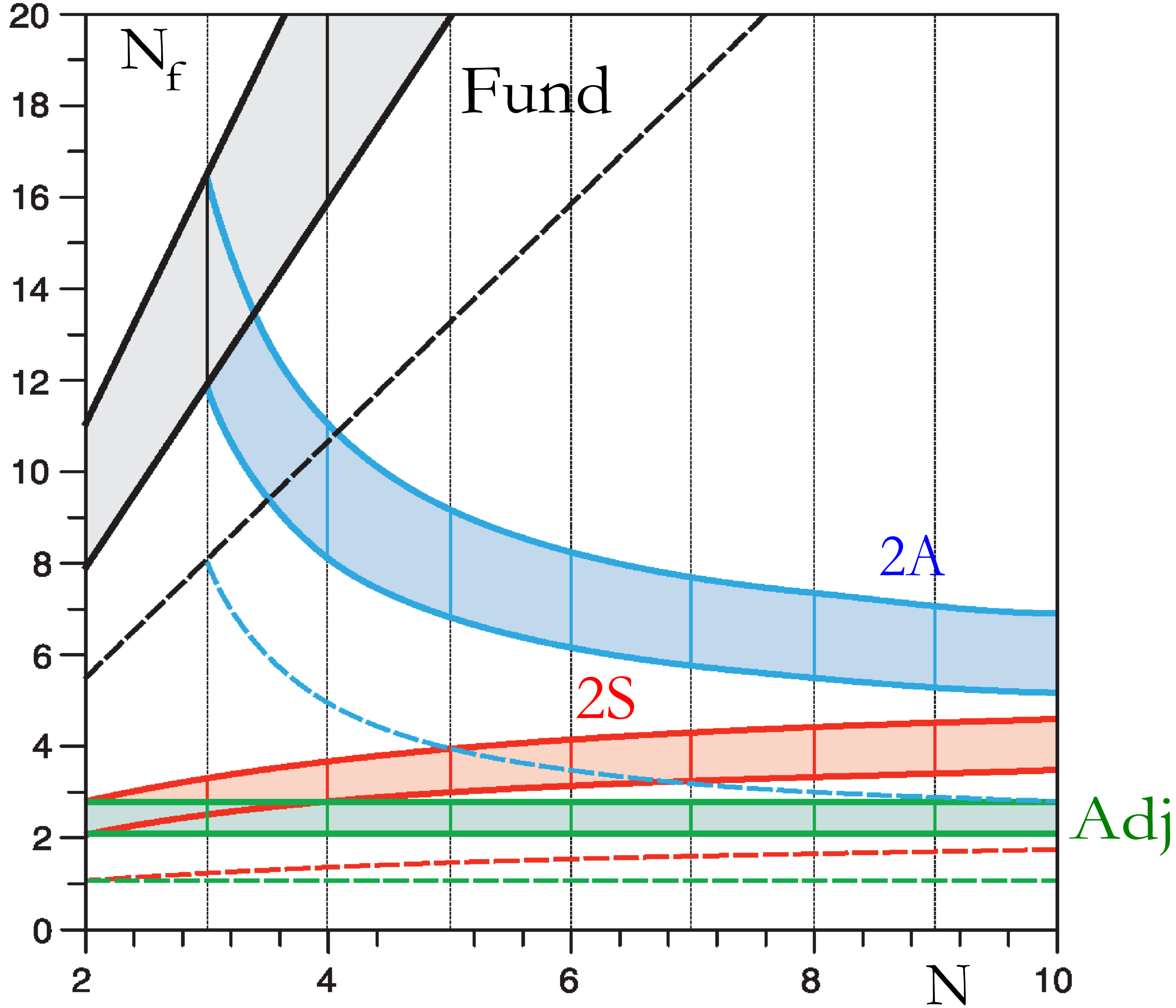


Figure credit: F. Sannino

Operator formalism and the Hamiltonian

- Operator formaliam of the Thirring model Hamiltonian

C.R. Hagen, 1967

$$H_{\mathrm{Th}} = \int dx \left[-i\bar{\psi}\gamma^1\partial_1\psi + m_0\bar{\psi}\psi + \frac{g}{4}\left(\bar{\psi}\gamma^0\psi\right)^2 - \frac{g}{4}\left(1+\frac{2g}{\pi}\right)^{-1}\left(\bar{\psi}\gamma^1\psi\right)^2 \right]$$

- Staggering, J-W transformation ($S_j^{\pm} = S_j^x \pm i S_j^y$):

J. Kogut and L. Susskind, 1975; A. Luther, 1976

$$\bar{H}_{XXZ} = \nu(g)\left[-\frac{1}{2}\sum_n^{N-2}\left(S_n^+ S_{n+1}^- + S_{n+1}^+ S_n^-\right) + a\tilde{m}_0 \sum_n^{N-1}(-1)^n\left(S_n^z + \frac{1}{2}\right) + \Delta(g)\sum_n^{N-1}\left(S_n^z+\frac{1}{2}\right)\left(S_{n+1}^z+\frac{1}{2}\right)\right]$$

$$\nu(g) = \frac{2\gamma}{\pi\sin(\gamma)},\ \tilde{m}_0 = \frac{m_0}{\nu(g)},\ \Delta(g) = \cos(\gamma),\ \text{with } \gamma = \frac{\pi - g}{2}$$

$$\boxed{\bar{H}_{\mathrm{sim}} = \frac{\bar{H}_{XXZ}}{\nu(g)} + \lambda\left(\sum_{n=0}^{N-1} S_n^z - \boxed{S_{\mathrm{target}}}\right)^2}$$

projected to a sector of total spin

JW-trans of the total fermion number,
Bosonise to topological index in the SG theory.

Simulation details for the phase structure

- Matrix product operator for the Hamiltonian (bulk)

$$W^{[n]} = \begin{pmatrix} 1_{2\times 2} & -\frac{1}{2}S^+ & -\frac{1}{2}S^- & 2\lambda S^z & \Delta S^z & \beta_n S^z + \alpha 1_{2\times 2} \\ 0 & 0 & 0 & 0 & 0 & S^- \\ 0 & 0 & 0 & 0 & 0 & S^+ \\ 0 & 0 & 0 & 1 & 0 & S^z \\ 0 & 0 & 0 & 0 & 0 & S^z \\ 0 & 0 & 0 & 0 & 0 & 1_{2\times 2} \end{pmatrix}$$

$$\beta_n = \Delta + (-1)^n \tilde{m}_0 a - 2\lambda\, S_{\text{target}}\,,\; \alpha = \lambda \left(\frac{1}{4} + \frac{S^2_{\text{target}}}{N} \right) + \frac{\Delta}{4}$$

- Simulation parameters

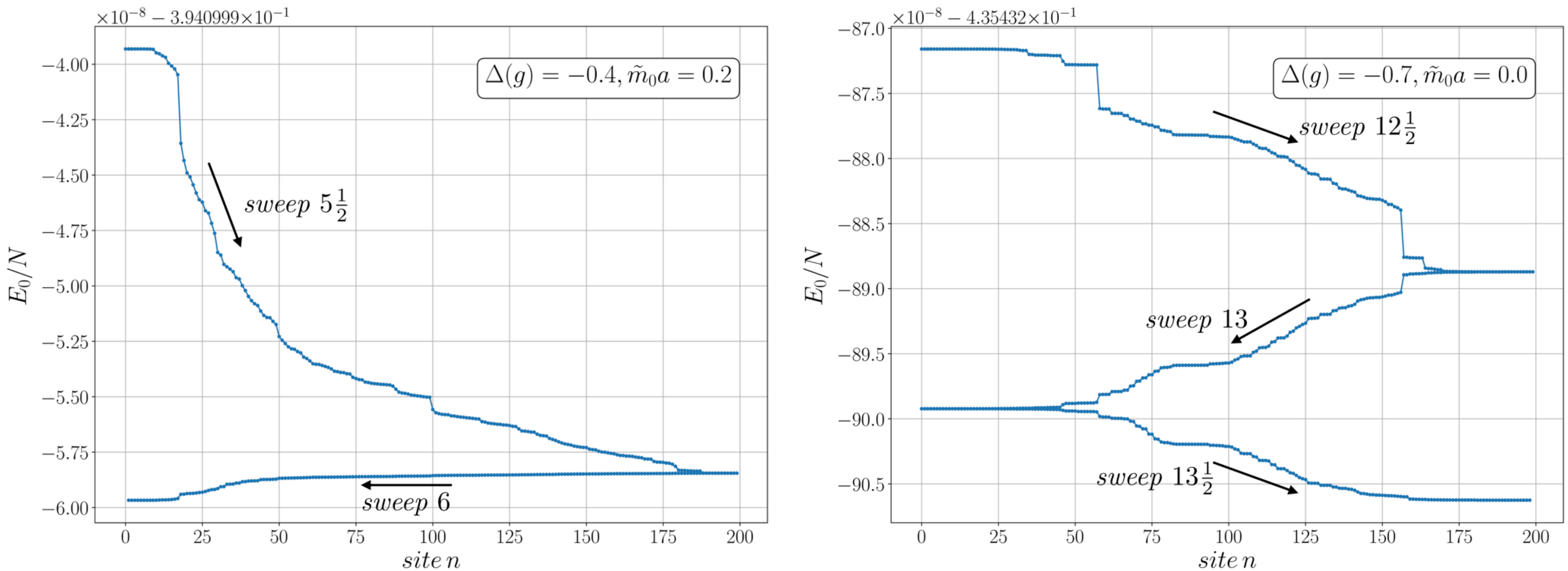
  ★ Twenty values of $\Delta(g)$, ranging from -0.9 to 1.0

  ★ Fourteen values of $\tilde{m}_0 a$, ranging from 0 to 0.4

  ★ Bond dimension $D = 50, 100, 200, 300, 400, 500, 600$

  ★ System size $N = 400, 600, 800, 1000$

Convergence

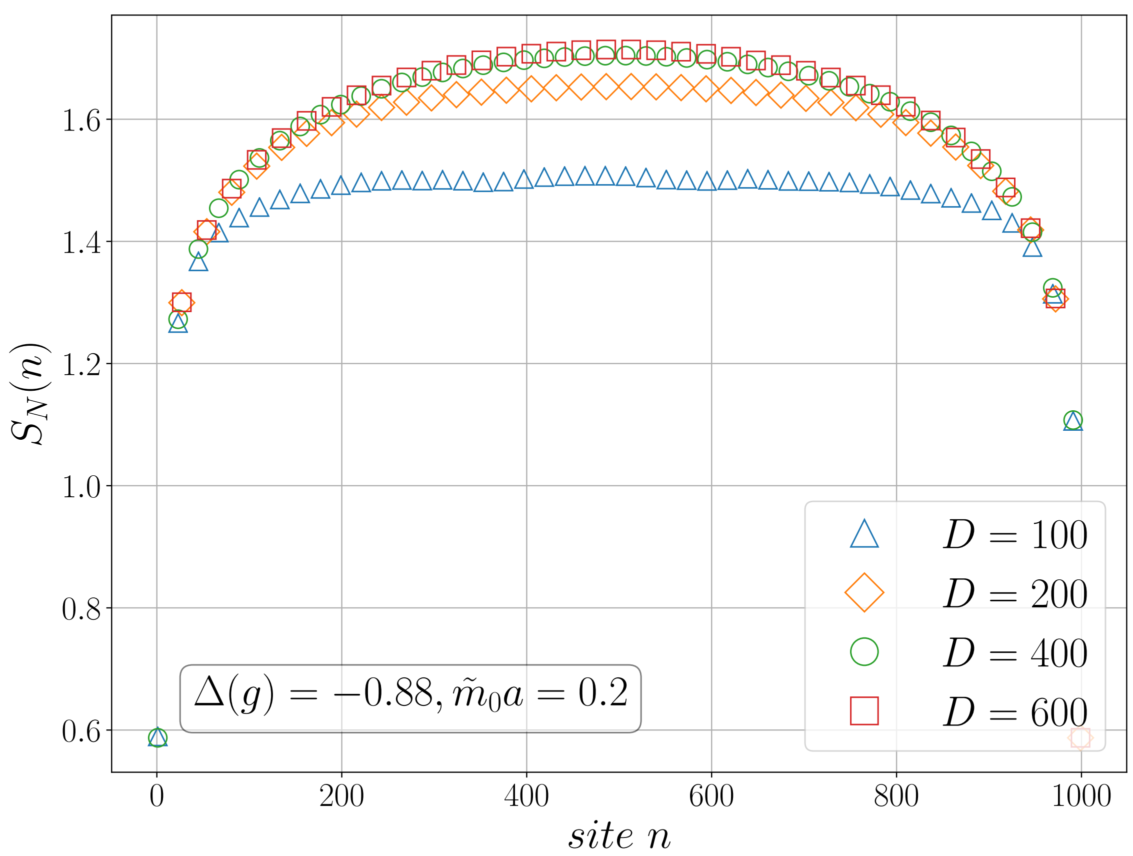


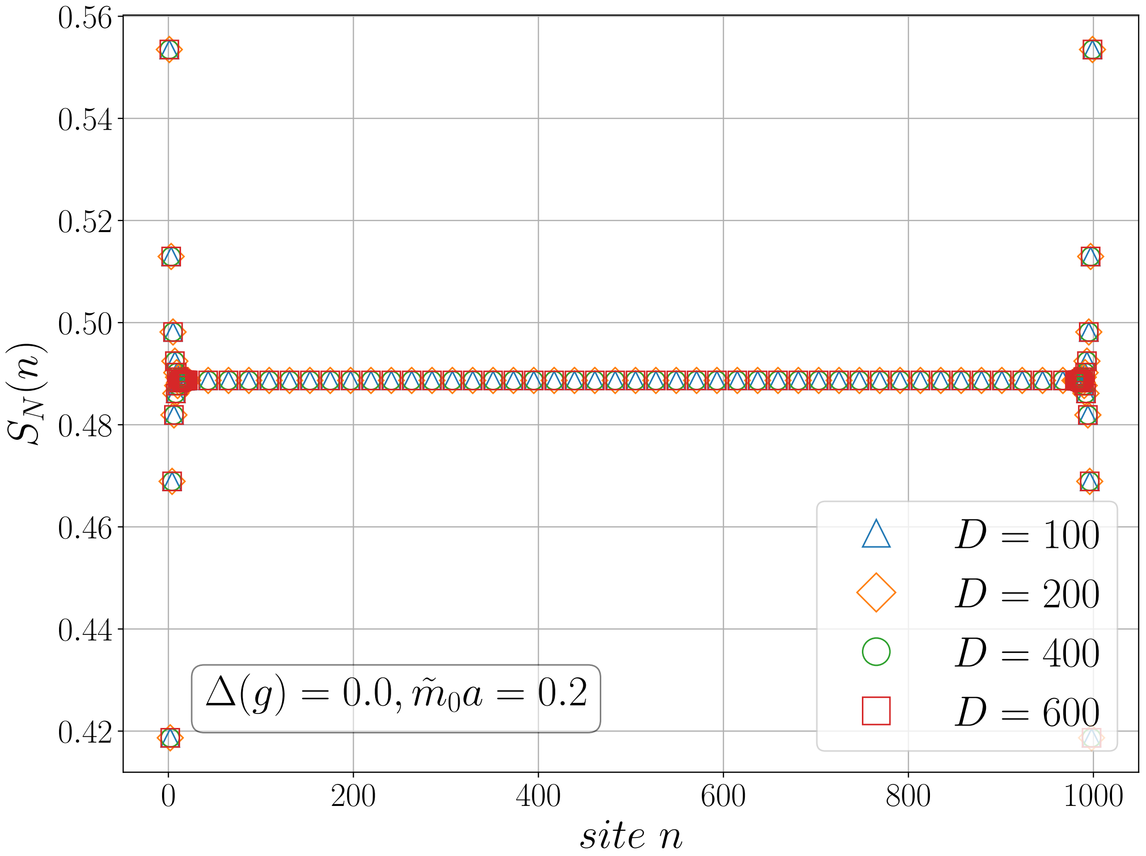
★ different convergence properties observed

Entanglement entropy

Calabrese-Cardy scaling and the central charge

$$S_N(n) = \frac{c}{6}\,\ln\left[\frac{N}{\pi}\sin\left(\frac{\pi n}{N}\right)\right] + k$$





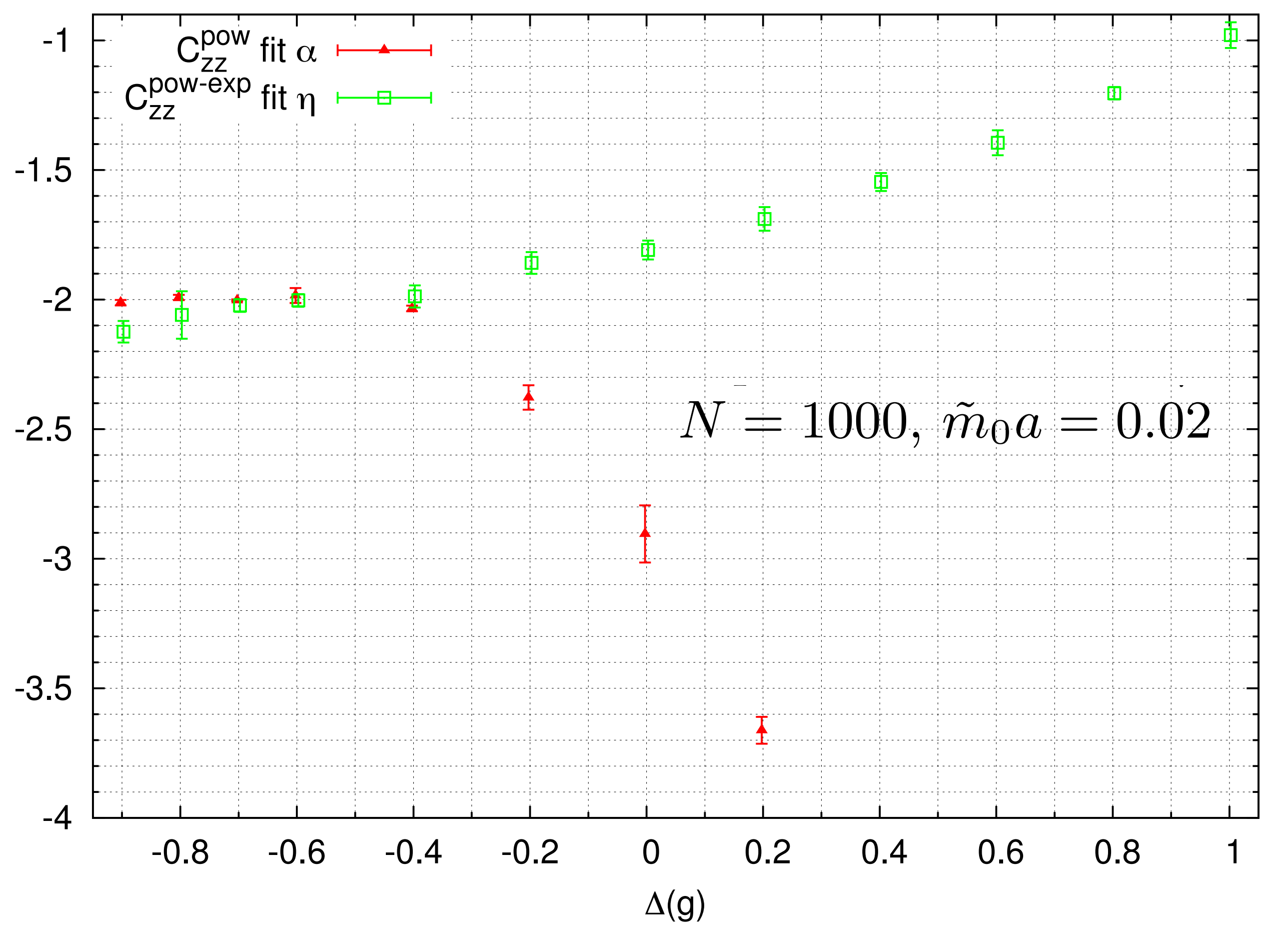
- ★ Scaling observed at $\Delta(g) \lesssim -0.7$ for $\tilde{m}_0 a \neq 0$, and for all values of $\Delta(g)$ at $\tilde{m}_0 a = 0$
- ★ In the critical phase, $c = 1$

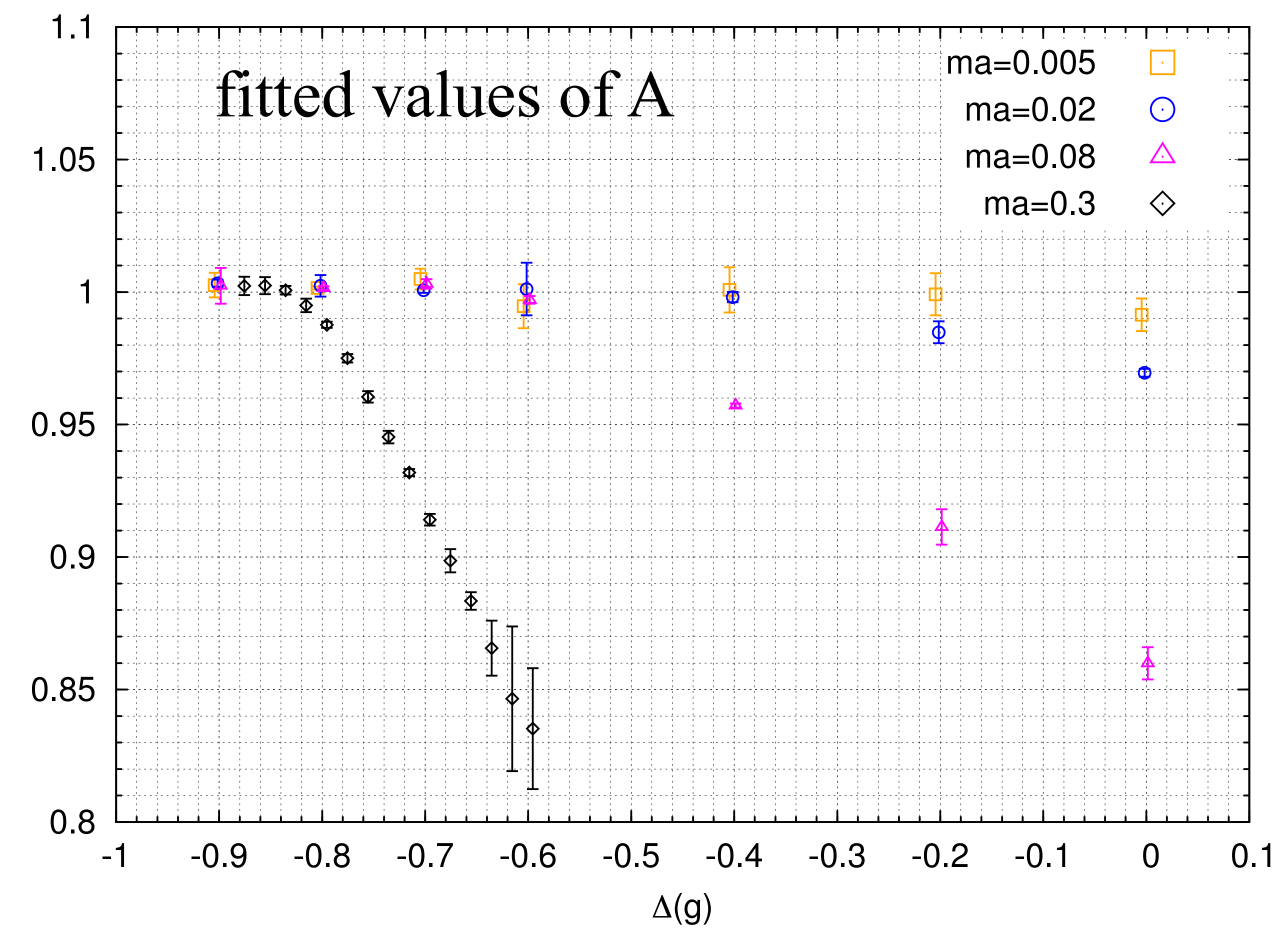
Density-density correlators

$$C_{zz}(x) = \langle \bar{\psi}\psi(x_0+x)\bar{\psi}\psi(x_0)\rangle_{\text{conn}} \xrightarrow{\text{JW trans}} \frac{1}{N_x}\sum_n S^z(n)S^z(n+x) - \frac{1}{N_0}\sum_n S^z(n)\sum_n S^z(n+1)$$

try fitting to

$$C_{zz}^{\text{pow}}(x) = \beta x^{\alpha} \text{ and } C_{zz}^{\text{pow-exp}}(x) = Bx^{\eta}A^{x}$$





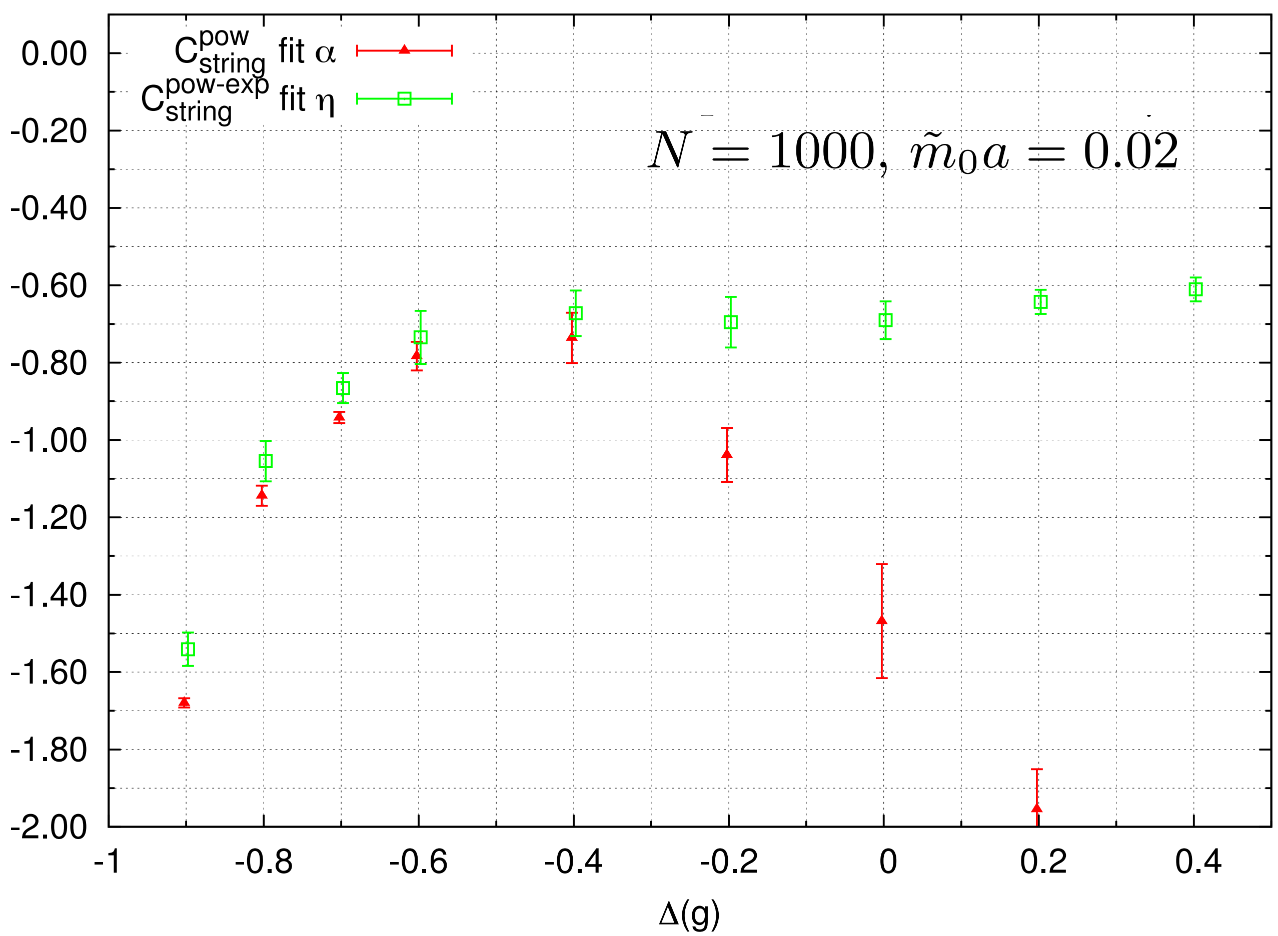
★ Evidence for a critical phase

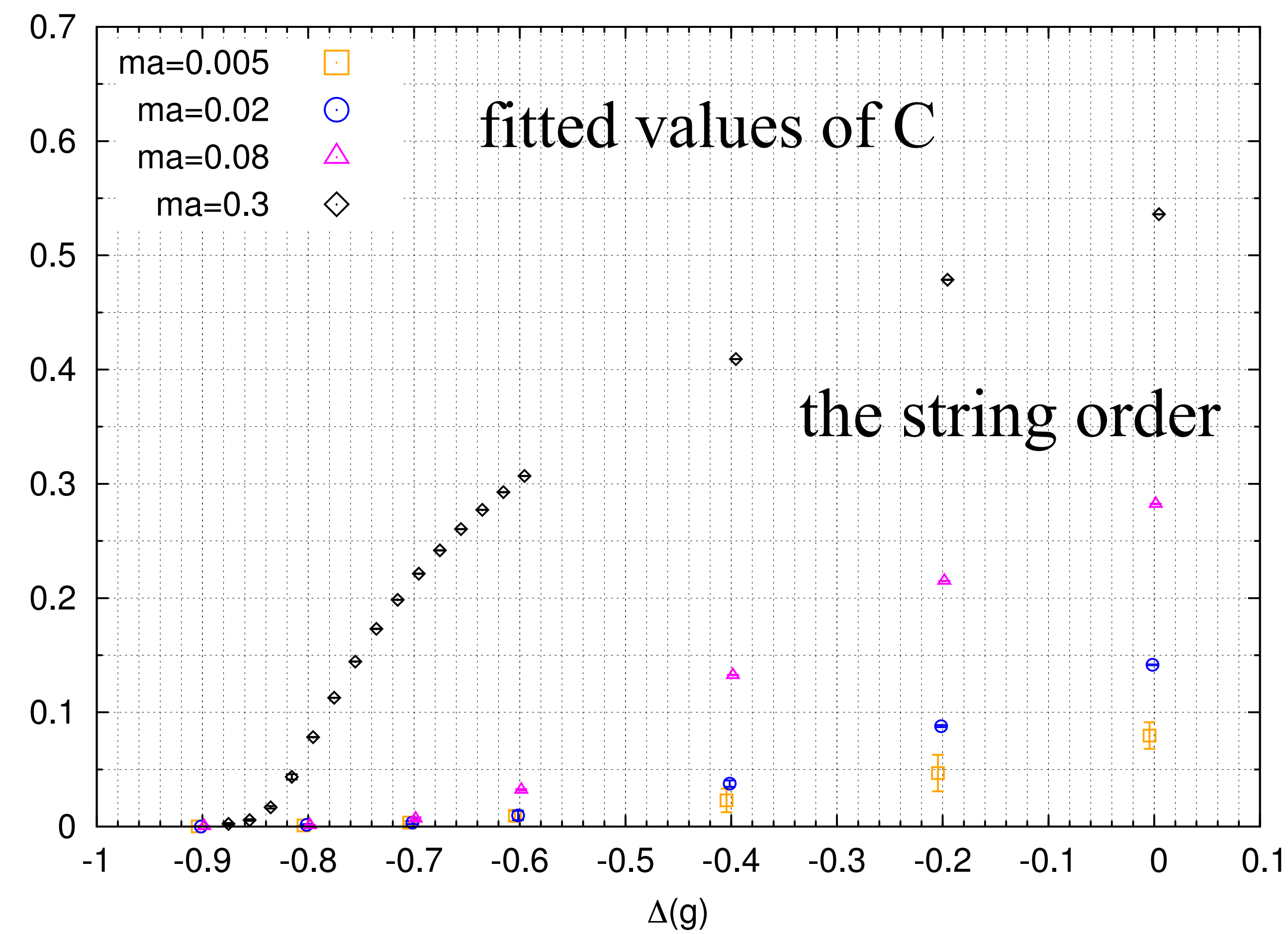
Soliton (string) correlators

$$C_{\text{string}}(x) = \langle \psi^\dagger(x_0+x)\psi(x_0)\rangle \xrightarrow{\text{JW trans}} \frac{1}{N_x}\sum_n S^+(n)S^z(n+1)\cdots S^z(n+x-1)S^-(n+x)$$

try fitting to

$$C^{\text{pow}}_{\text{string}}(x) = \beta x^\alpha + C \quad \text{and} \quad C^{\text{pow-exp}}_{\text{string}}(x) = Bx^\eta A^x + C$$

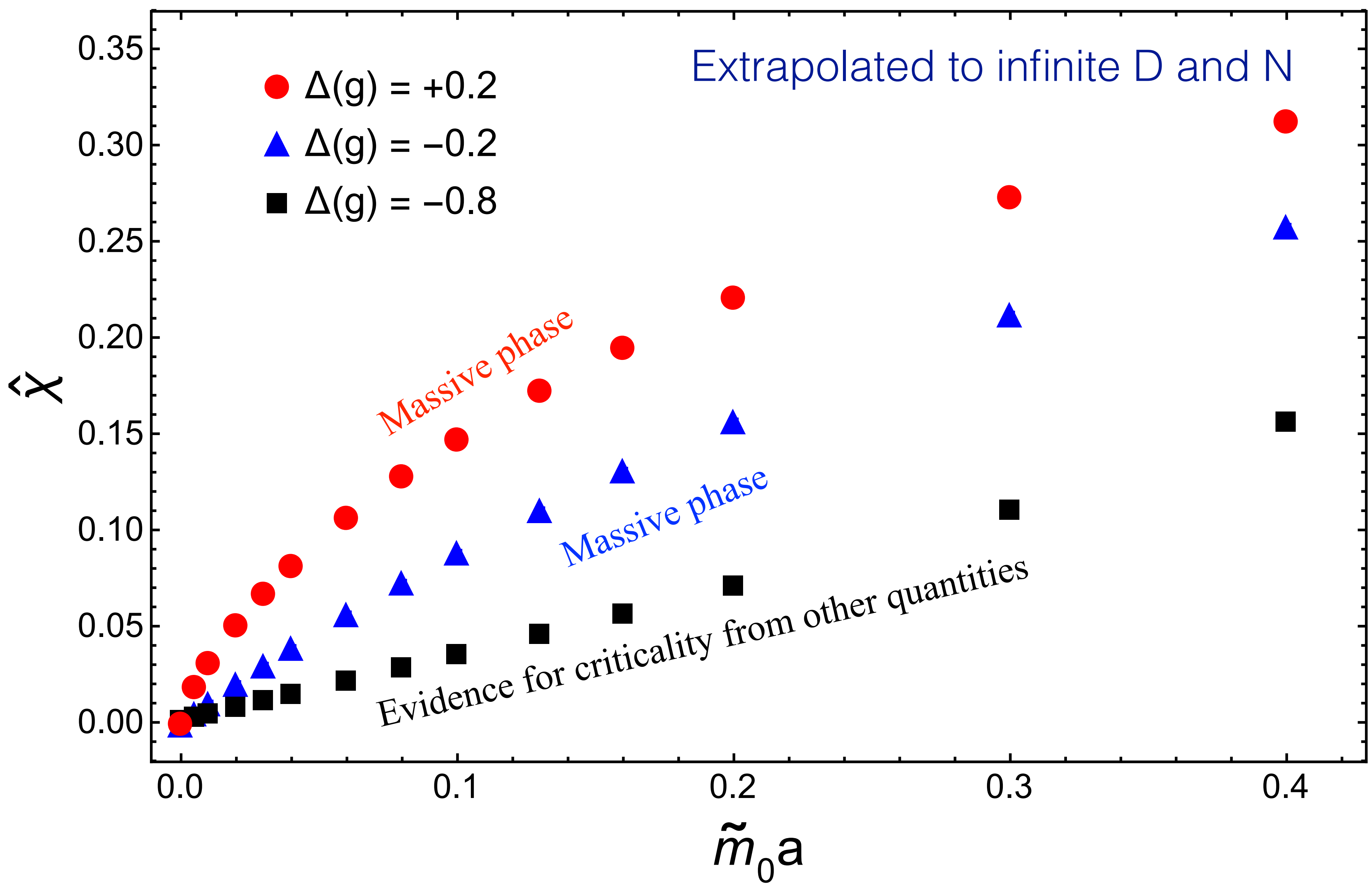




★ Similar behaviour in A. Evidence for a critical phase

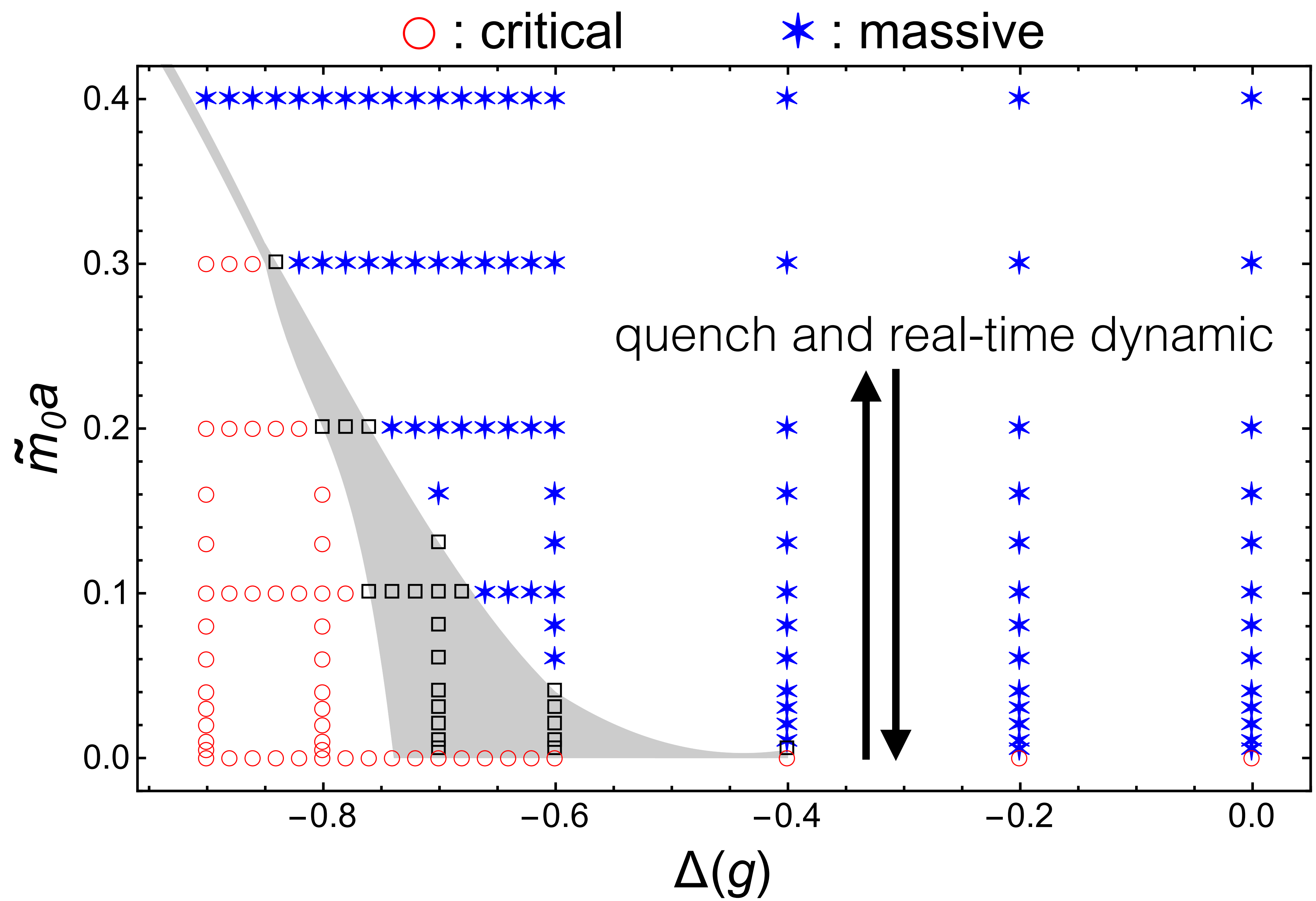
Chiral condensate

$$\hat{\chi} = a\left|\langle\bar{\psi}\psi\rangle\right| = \frac{1}{N}\left|\sum_n (-1)^n S_n^z\right|$$



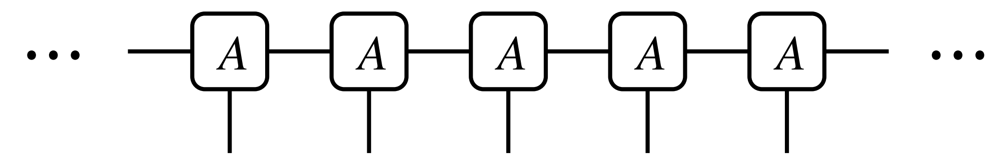
★ Chiral condensate is not an order parameter

Phase structure of the Thirring model



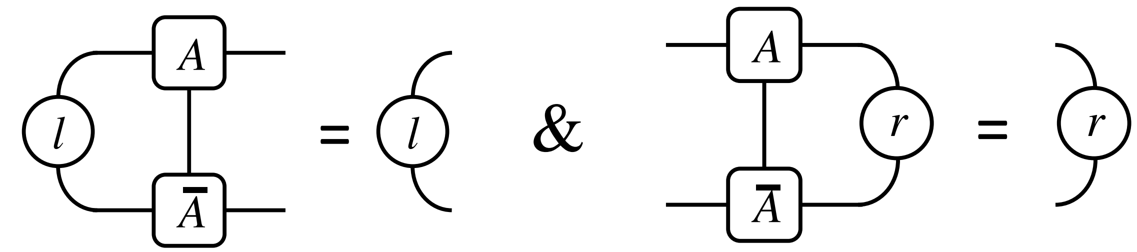
Uniform MPS and real-time evolution

★ Translational invariance in MPS



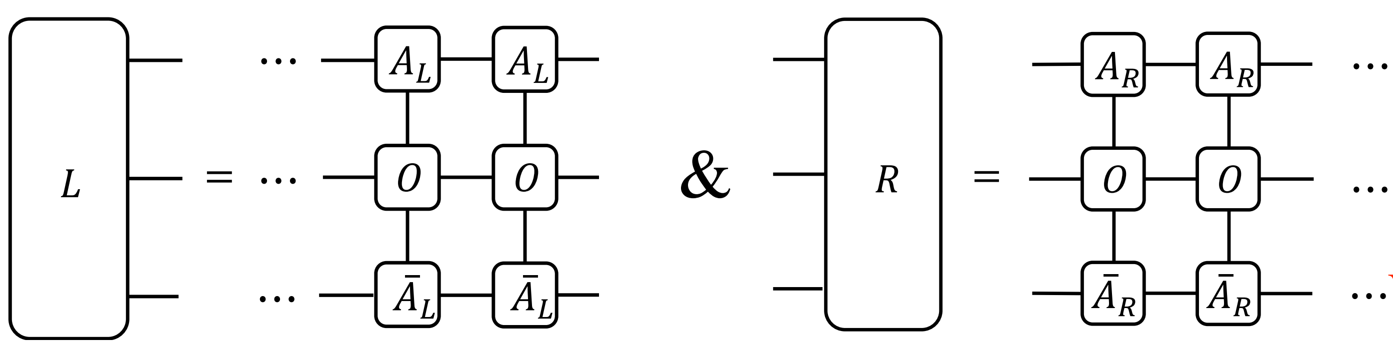
★ Finding the infinite BC for amplitudes

(largest eigenvalue normalised to be 1)



H.N. Phien, G. Vidal and I.P. McCulloch, Phys. Rev. B86, 2012

★ Similar (more complicated) procedure in the variation search for the ground state



V. Zauner-Stauber *et al*, Phys. Rev. B97, 2018

★ Real-time evolution *via* time-dependent variational principle

➡ Key: projection to MPS in $i\frac{d}{dt}|\Psi(A(t))\rangle = P_{|\Psi(A)\rangle}\hat{H}|\Psi(A(t))\rangle$

J. Haegeman *et al*, Phys. Rev. Lett.107, 2011

Dynamical quantum phase transition

★ "Quenching" : Sudden change of coupling strength in time evolution

$$H(g_1)|0_1\rangle = E_0^{(1)}|0_1\rangle \quad \text{and} \quad |\psi(t)\rangle = \mathrm{e}^{-iH(g_2)t}|0_1\rangle$$

★ Questions: Any singular behaviour? Related to equilibrium PT?

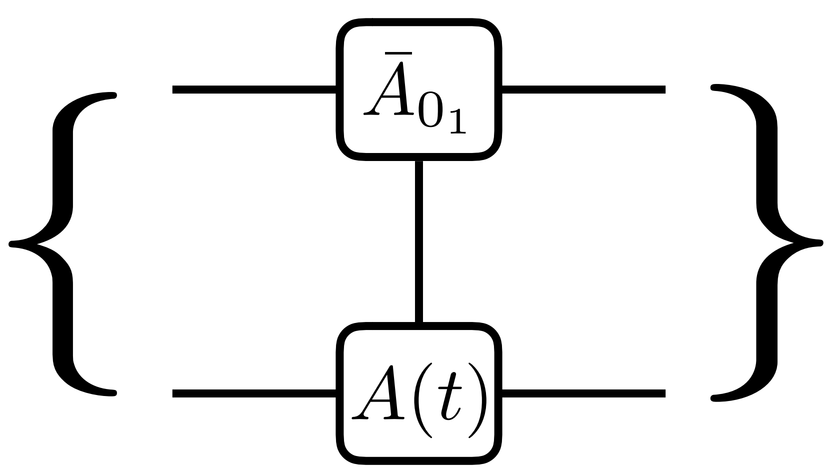
★ The Loschmidt echo and the return rate

$$L(t) = \langle 0_1|\mathrm{e}^{-iH(g_2)t}|0_1\rangle \qquad \& \qquad g(t) = -\lim_{N\to\infty}\frac{1}{N}\ln L(t)$$

➤ *c.f.*, the partition function and the free energy

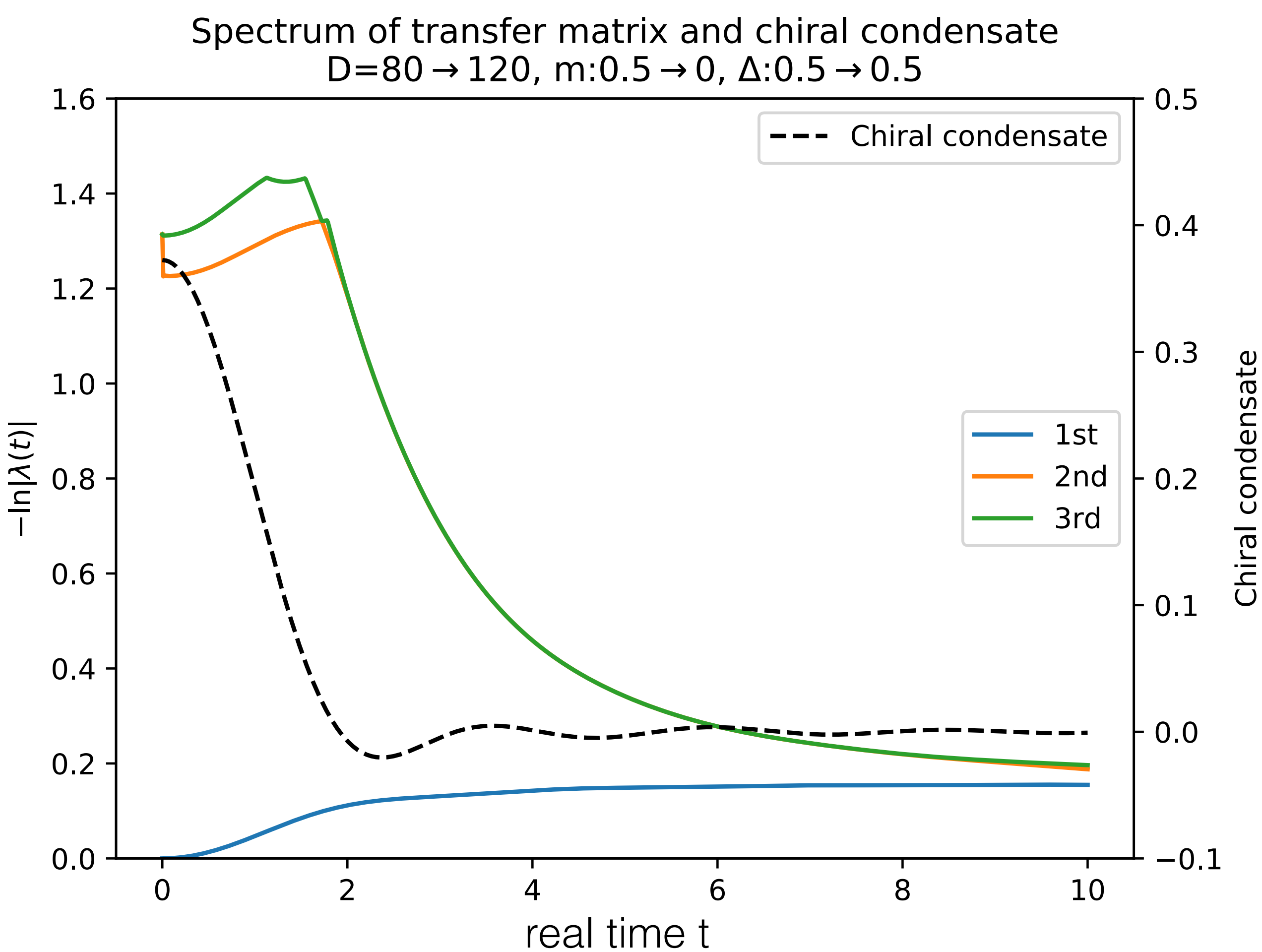
➤ In uMPS computed from the largest eigenvalue of the "transfer matrix"

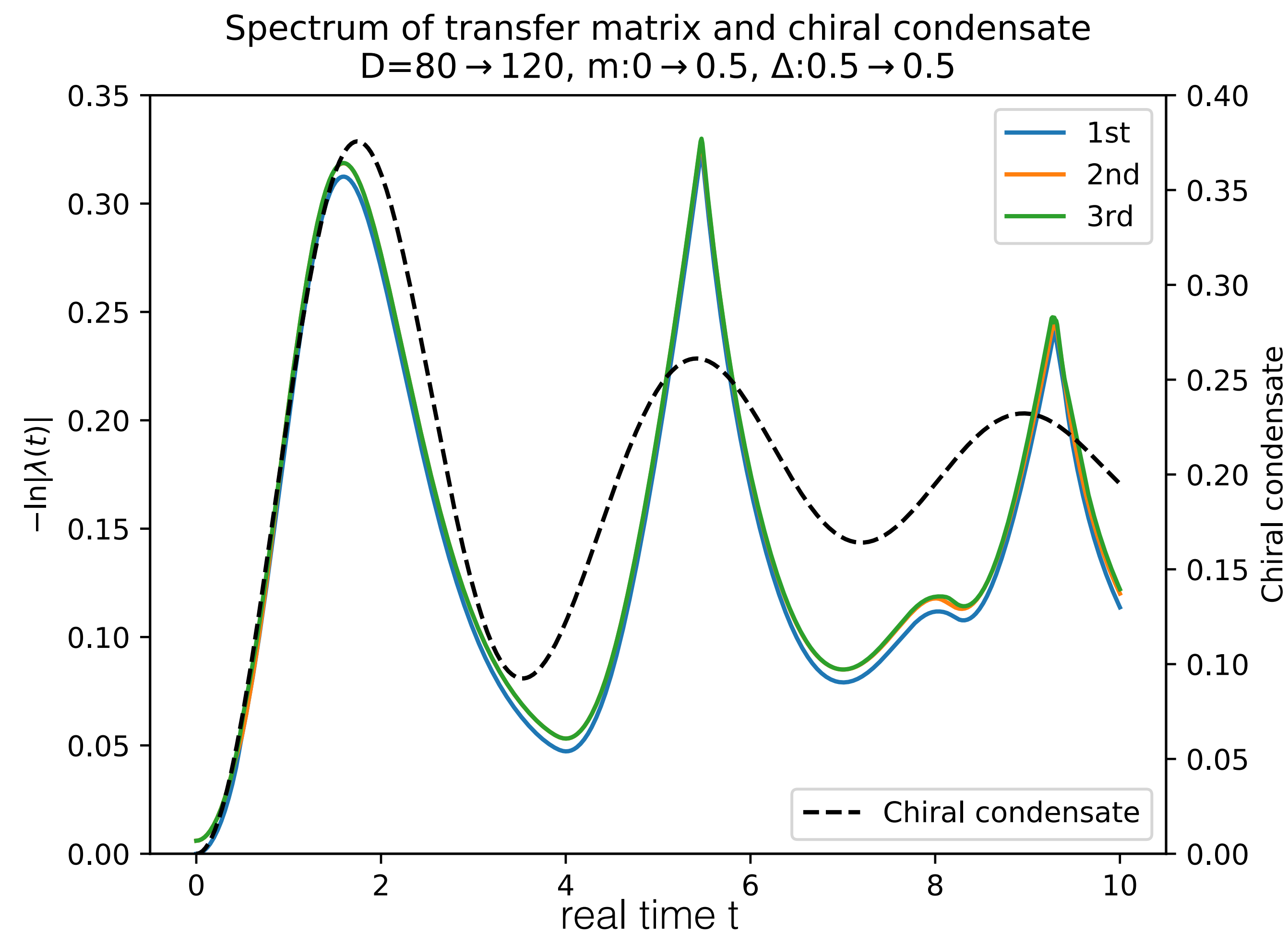
$T_{i,j}(t) = i$



j

Observing DQPT



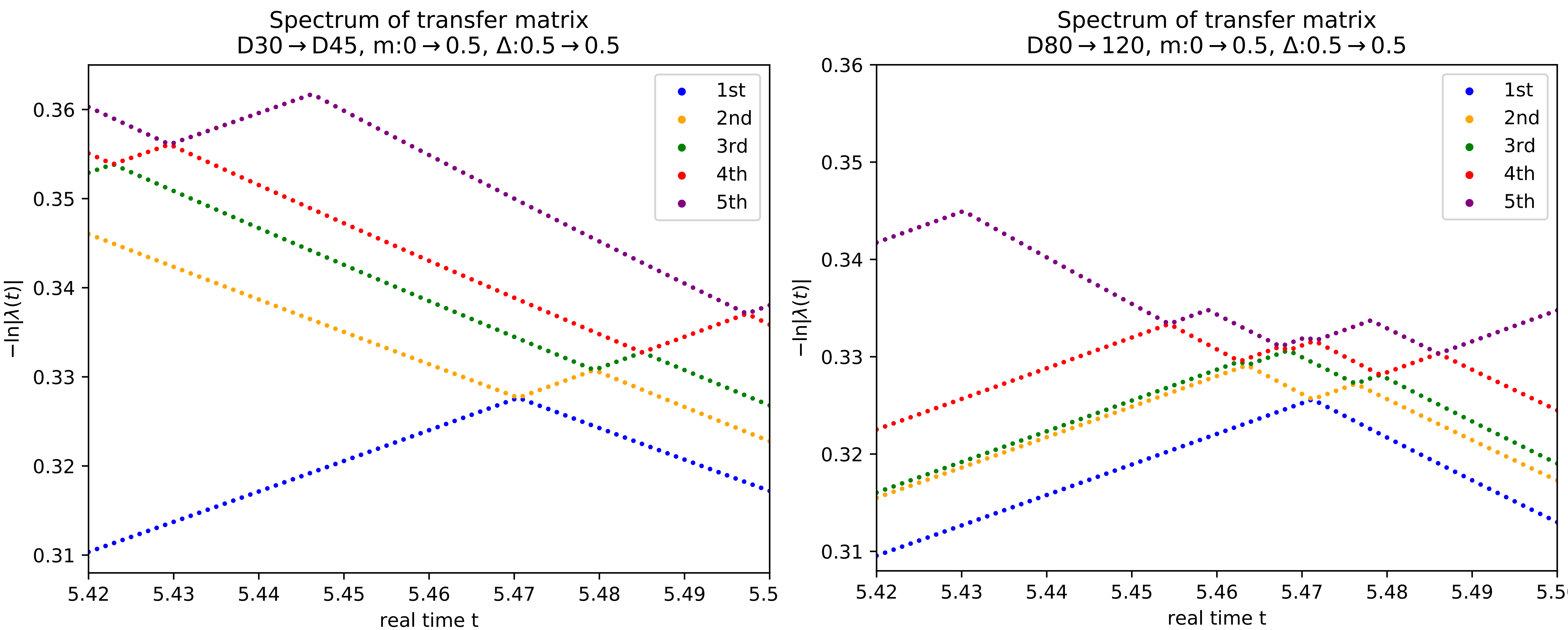


massive → critical

critical → massive

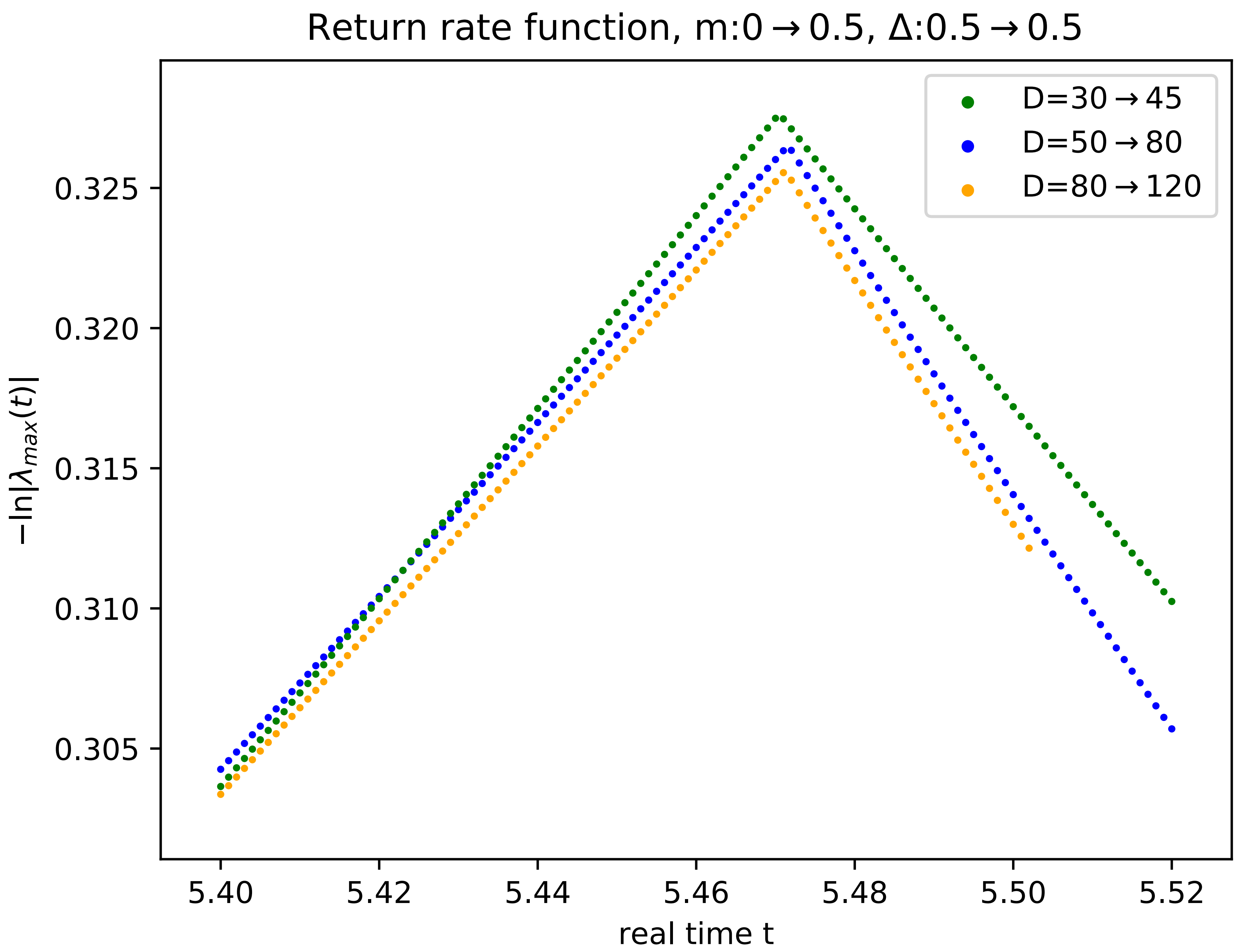
★ DQPT is a “one-way” transition…

DQPT and eigenvalue crossing



★ D-dependence in the crossing points

Bond-dim dependence in DQPT?



Conclusion and outlook

- Concluding results for phase structure
  - ★ KT-type transition in the massive Thirring model
- Exploratory results for real-time dynamics
  - ★ DQPT observed
  - ★ Relation to equilibrium KT phase transition?