New numerical approaches for directed percolation

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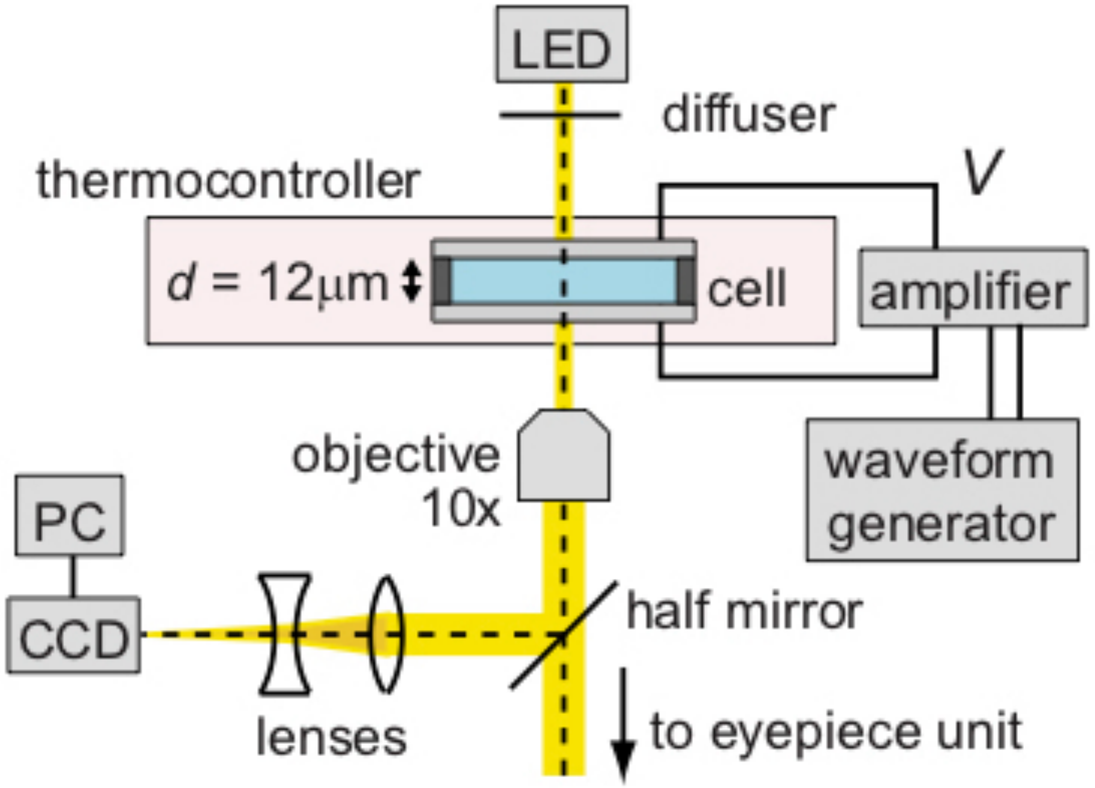
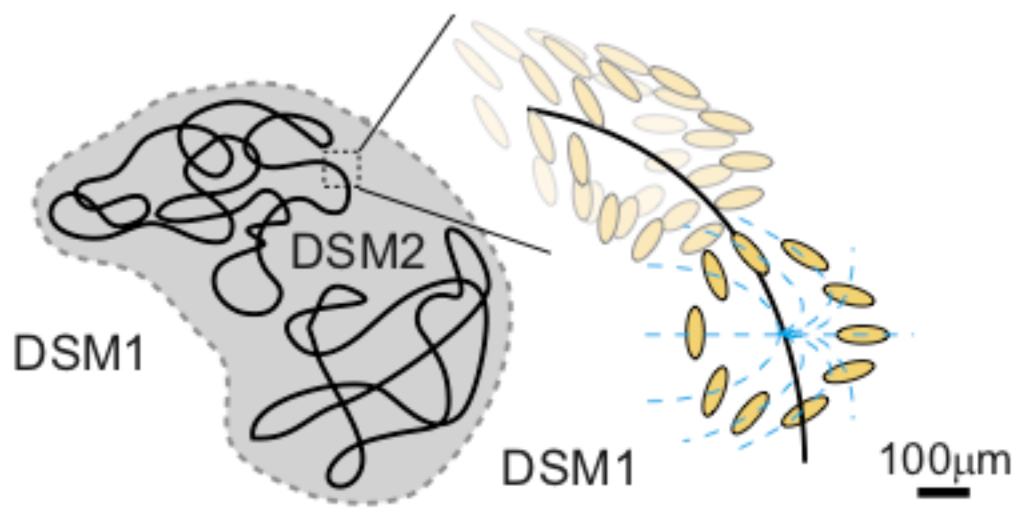
Turbulent liquid cristal

K.Z. Takeuchi, M. Kuroda, H. Chaté, M. Sano, PRL **99**, 234503 (2007)

Experimental setup

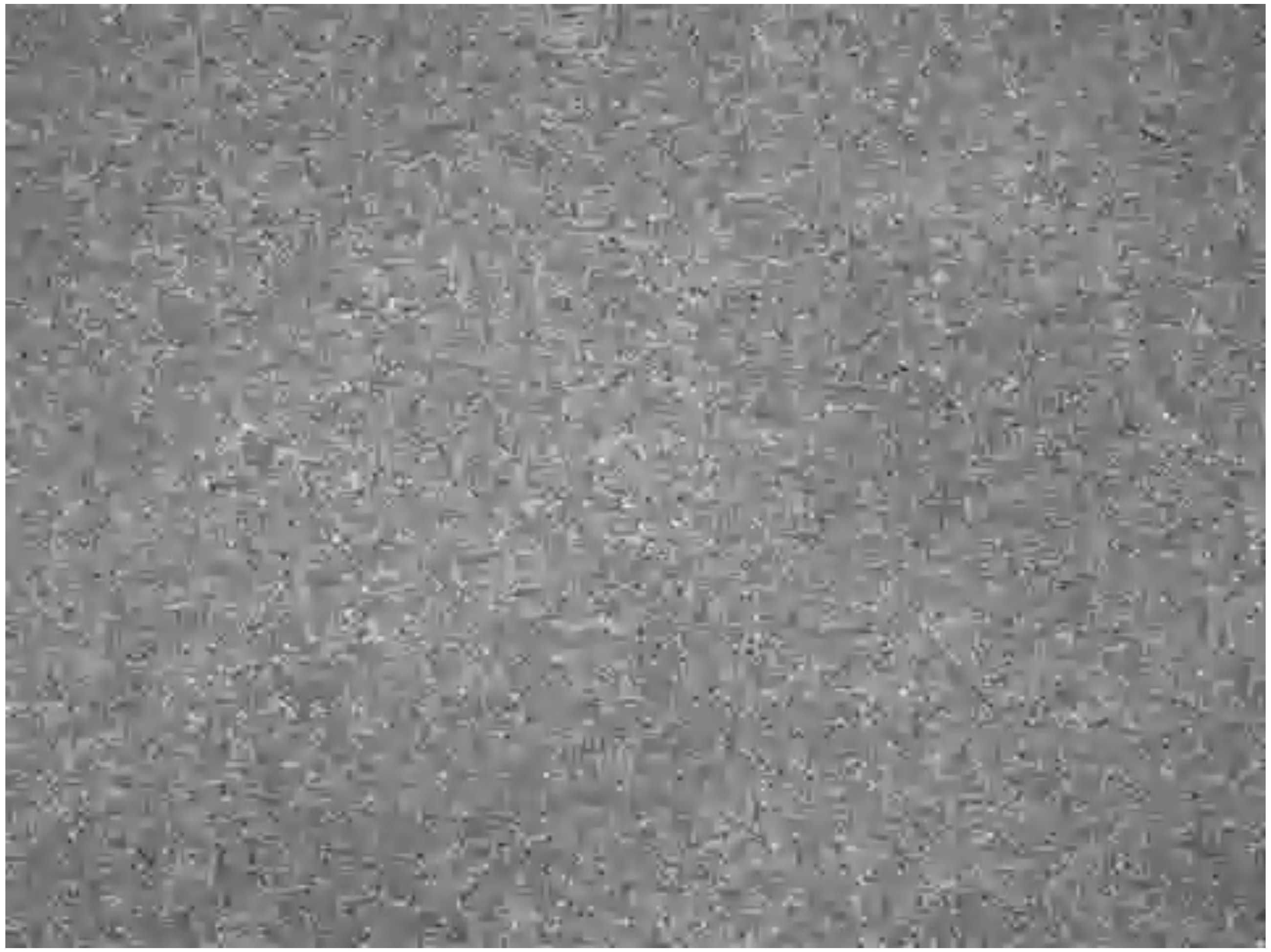


Dynamic scattering modes 1 & 2



DSM2 flat interface

on youtube



Absorbing state : no DSM2 site

K.Z. Takeuchi, M. Kuroda, H. Chaté, M. Sano, PRL **99**, 234503 (2007)

Dynamics of DSM2

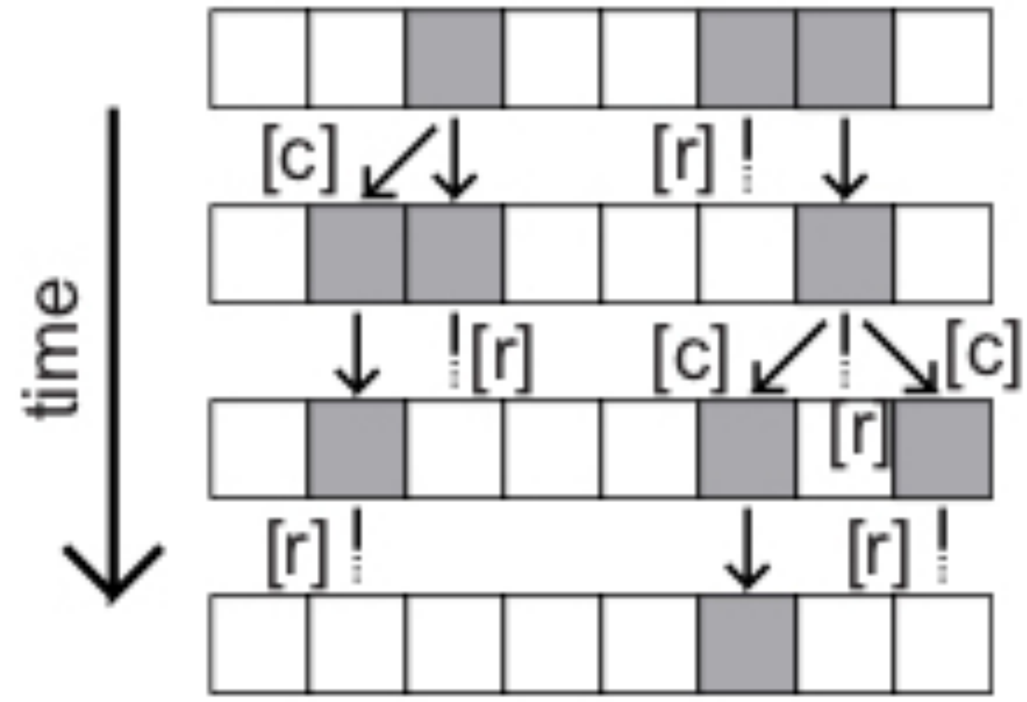
stochastically contaminate neighboring regions or relax
but no spontaneous nucleation

Non-equilibrium system

If there is no DSM2 site,
the system cannot escape

➡ **absorbing state**

➡ **no detail balance**

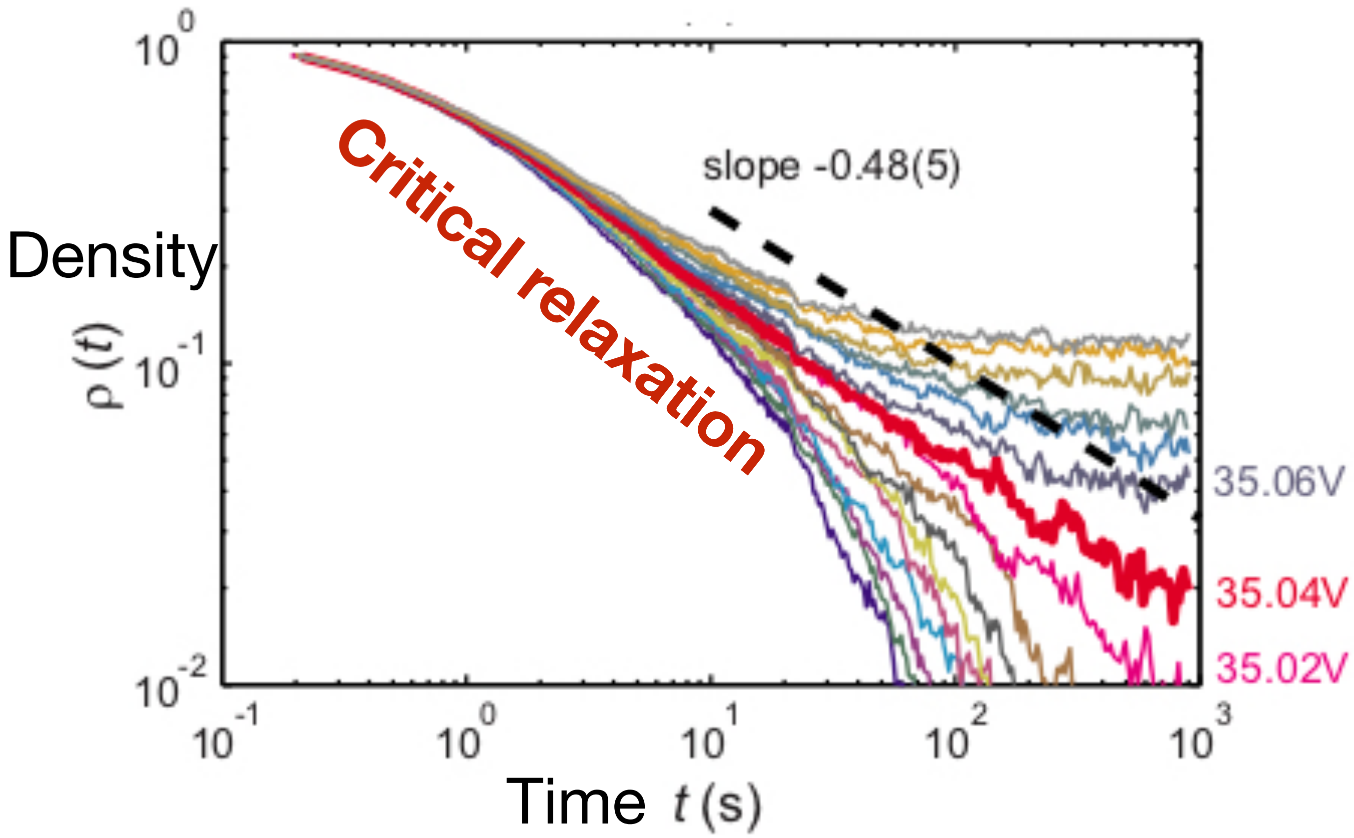


Directed percolation

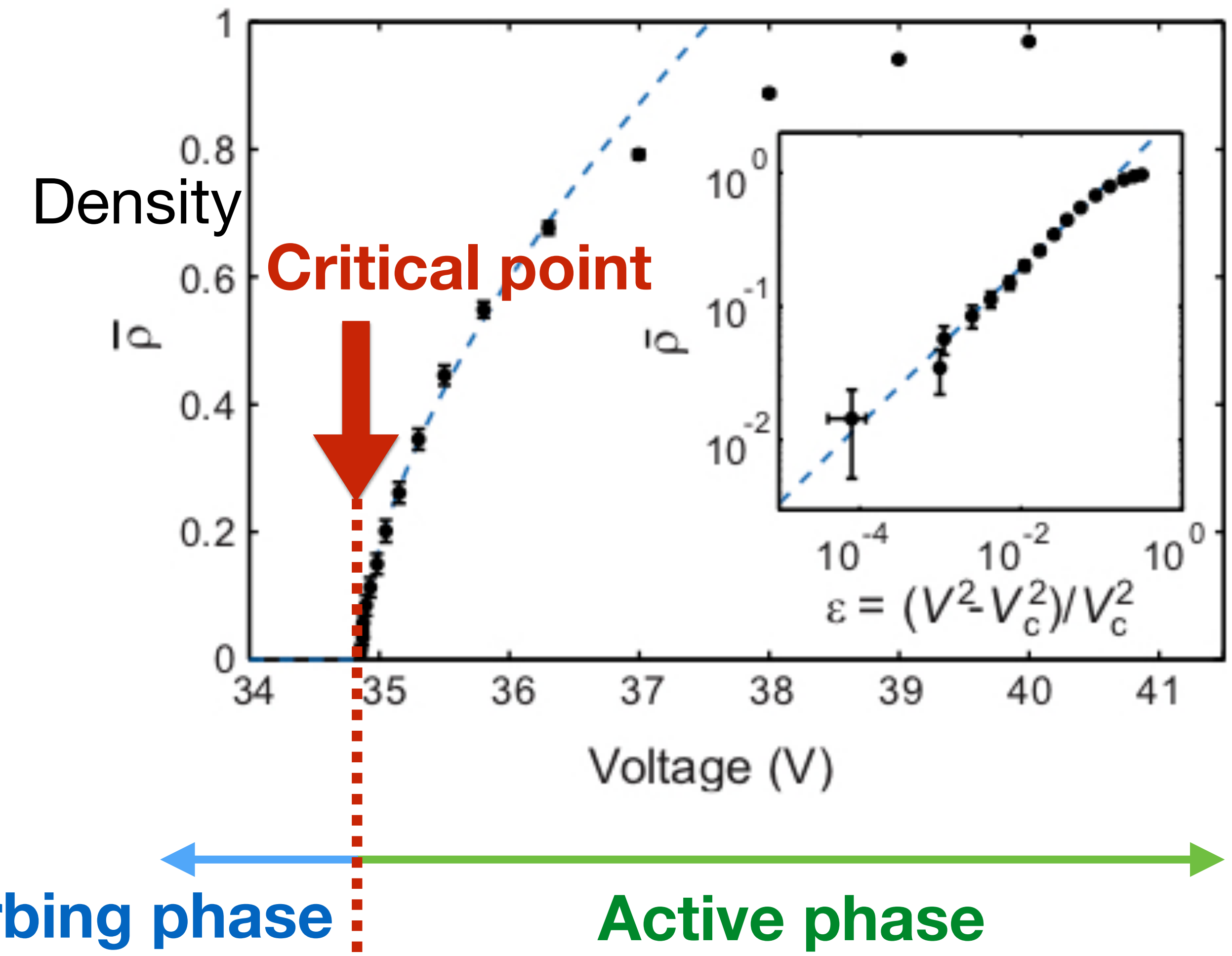
Criticality and universality of DP

K.Z. Takeuchi, M. Kuroda, H. Chaté, M. Sano, PRL **99**, 234503 (2007)

Decay of DSM2 density



DSM2 density in the steady state



DP universality class

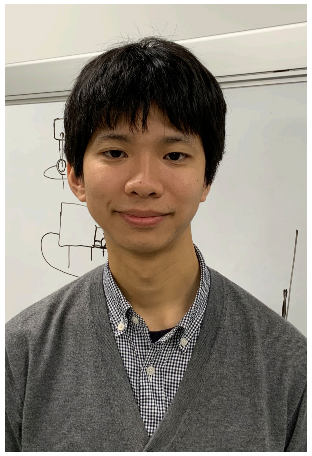
e.g. chemical reaction, turbulent, infectious disease, forest fire, …

The d-dim. reaction diffusion system of active objects = the (d+1)-dim. DP

[Ref] M. Henkel, H. Hinrichsen, and S. LÜBECK, Non-Equilibrium Phase Transitions. Volume 1: Absorbing Phase Transitions (Springer, 2008).

Outline

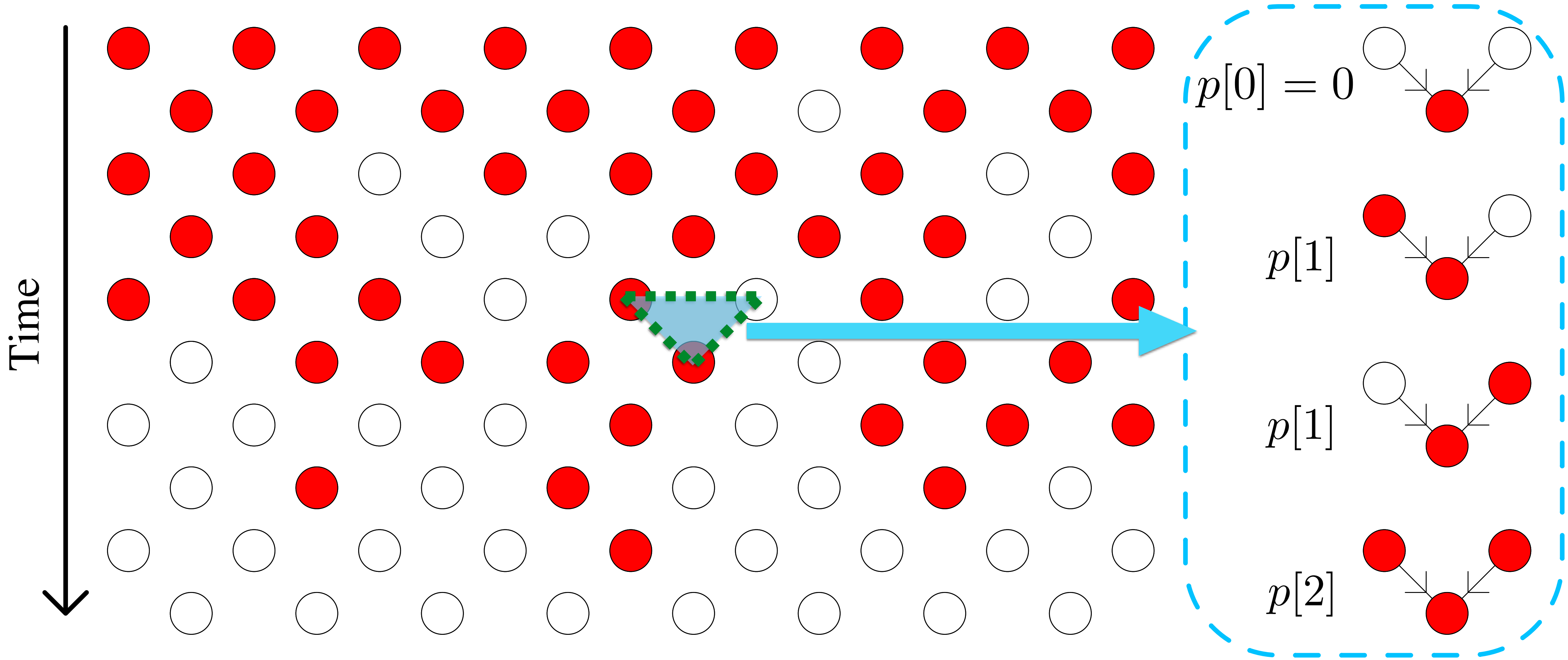
- **Domany-Kinzel automaton**
  Bond directed percolation
  Tensor network representation
- **Dynamical MPS simulation of one-dim. DK**
  Entanglement and Renyi entropies of DP
- **Monte Carlo calculation of double-DK**
  Free energy of d-DK = the second Renyi entropy of DP
- **TRG calculation for the transfer matrix of DK**
  Oblique projection
- **Discussion**



Collaboration with Naoki Kawashima, Yuki Hoshino, Kana Nataochi

Domany-Kinzel automaton

E. Domany and W. Kinzel, PRL **53**, 311 (1984).



Conditional probability = anisotropic three-body interaction

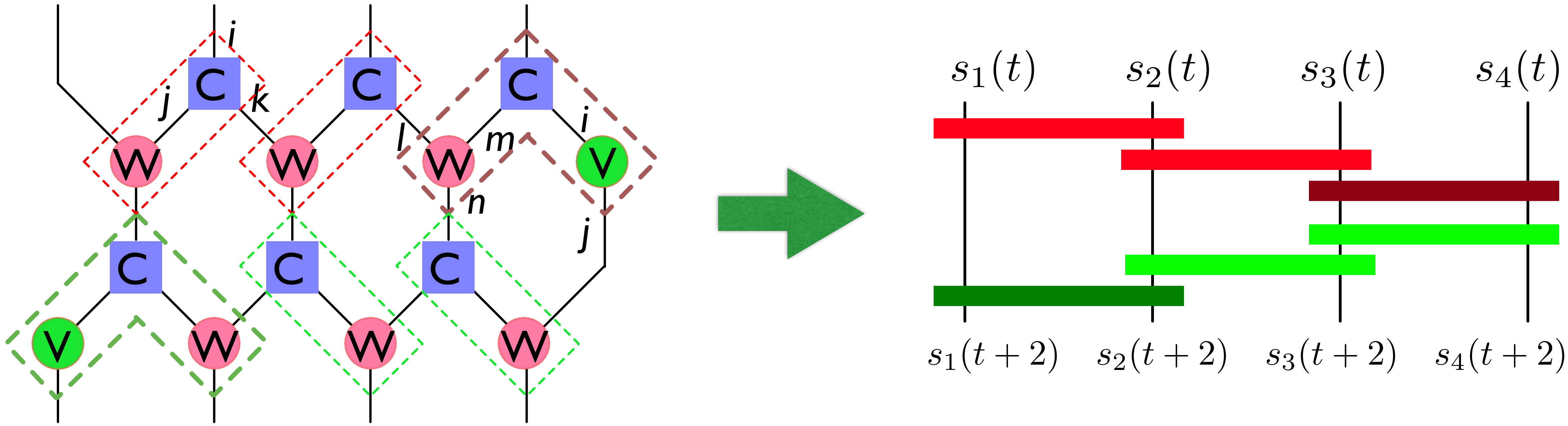
e.g. bond directed percolation

$$p[1] = p, \quad p[2] = 1 - (1-p)^2$$

Tensor network rep. of DK

Transfer matrix of DK

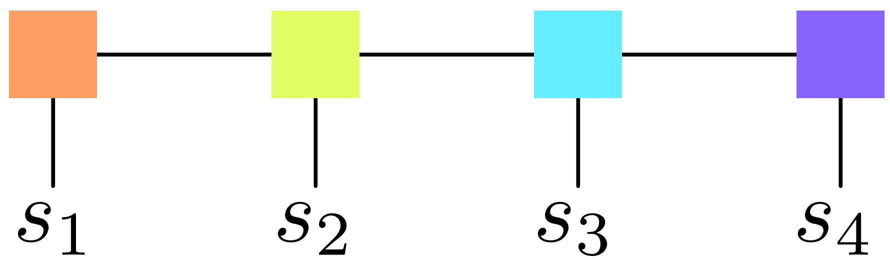
$$P\left(\vec{S}(t)\,\middle|\,\vec{S}(t-1)\right) = \prod_i P\left(S_i(t)|S_{i-1}(t-1), S_{i+1}(t-1)\right)$$



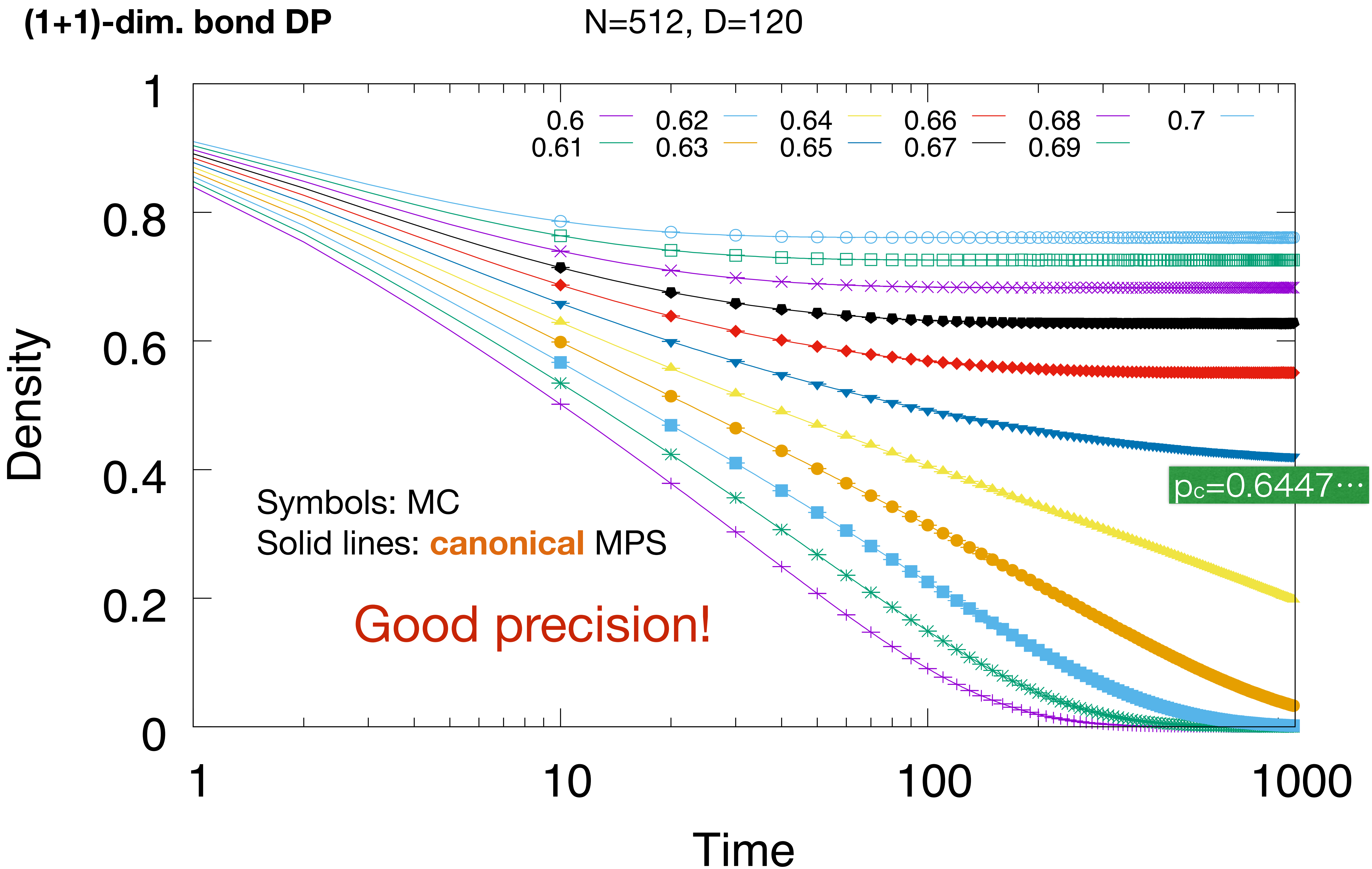
State probability distribution

e.g. MPS

$$P(s_1, s_2, s_3, s_4) \approx$$



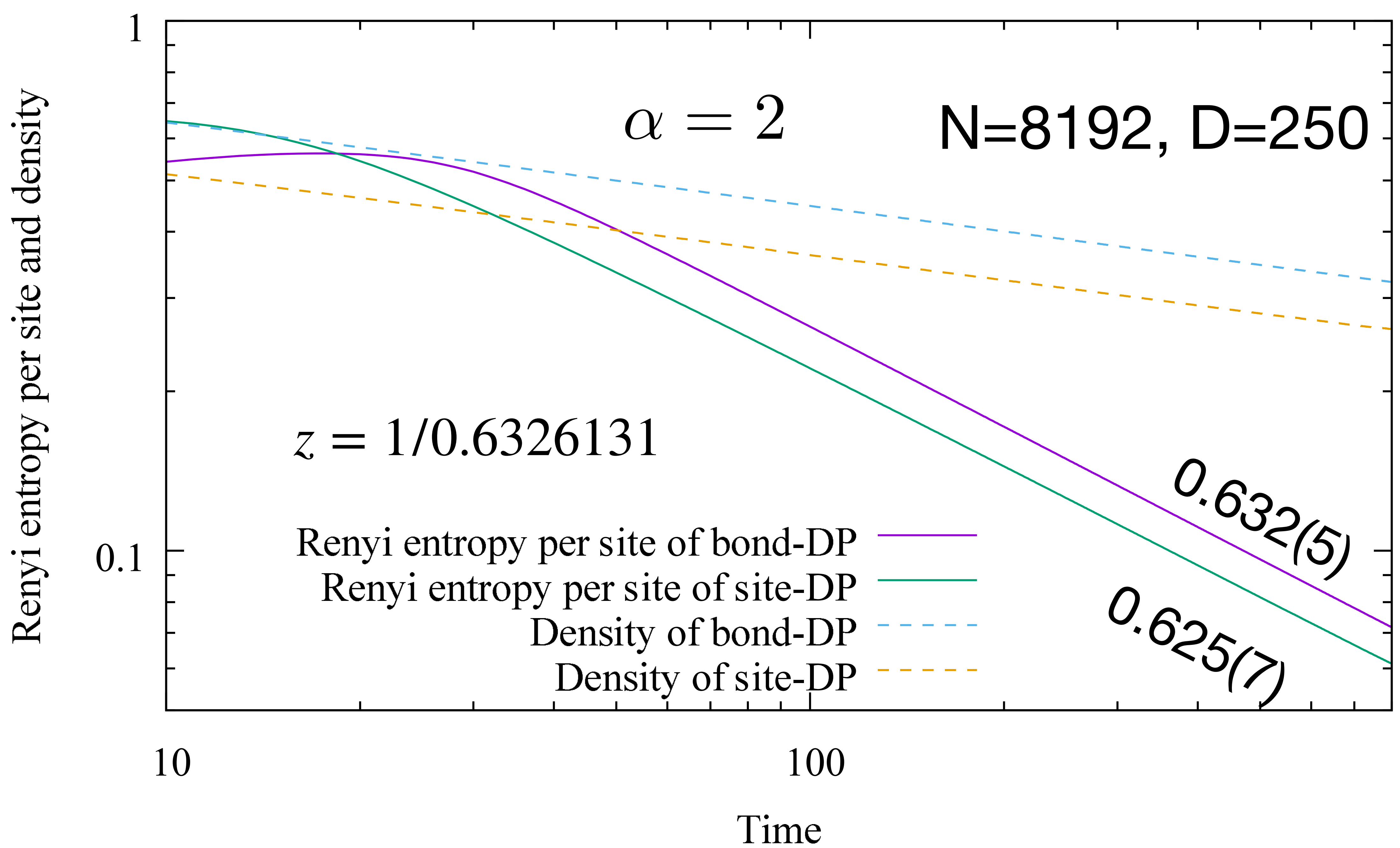
Quench dynamics of the active density



Critical relaxation of Renyi entropy at critical point

Harada & Kawashima, PRL **123**, 090601(2019).

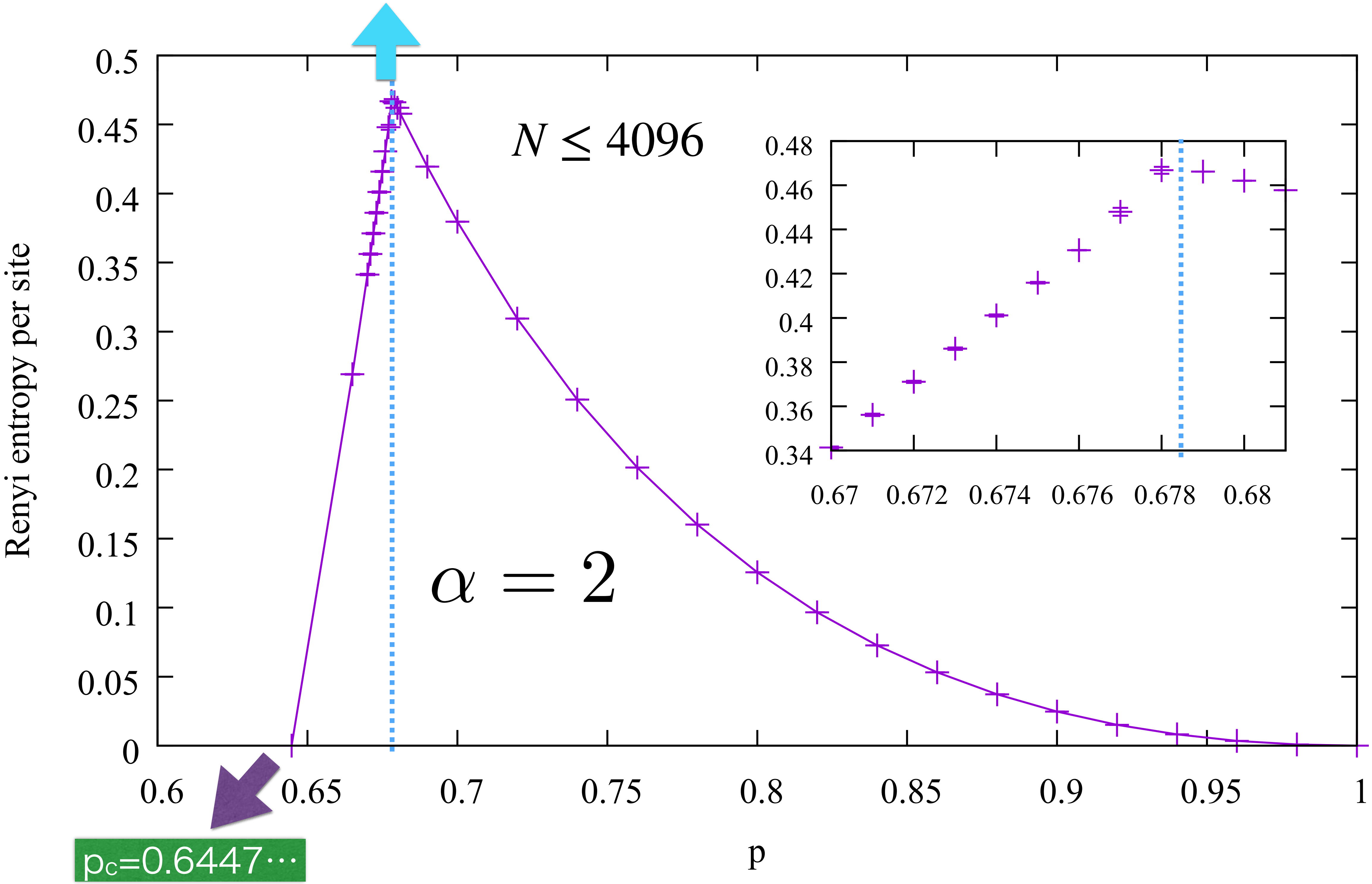
Renyi entropy $$H_\alpha = \frac{1}{1-\alpha} \log \sum_S P(S)^\alpha$$



New singularity of Renyi entropy of steady state distribution

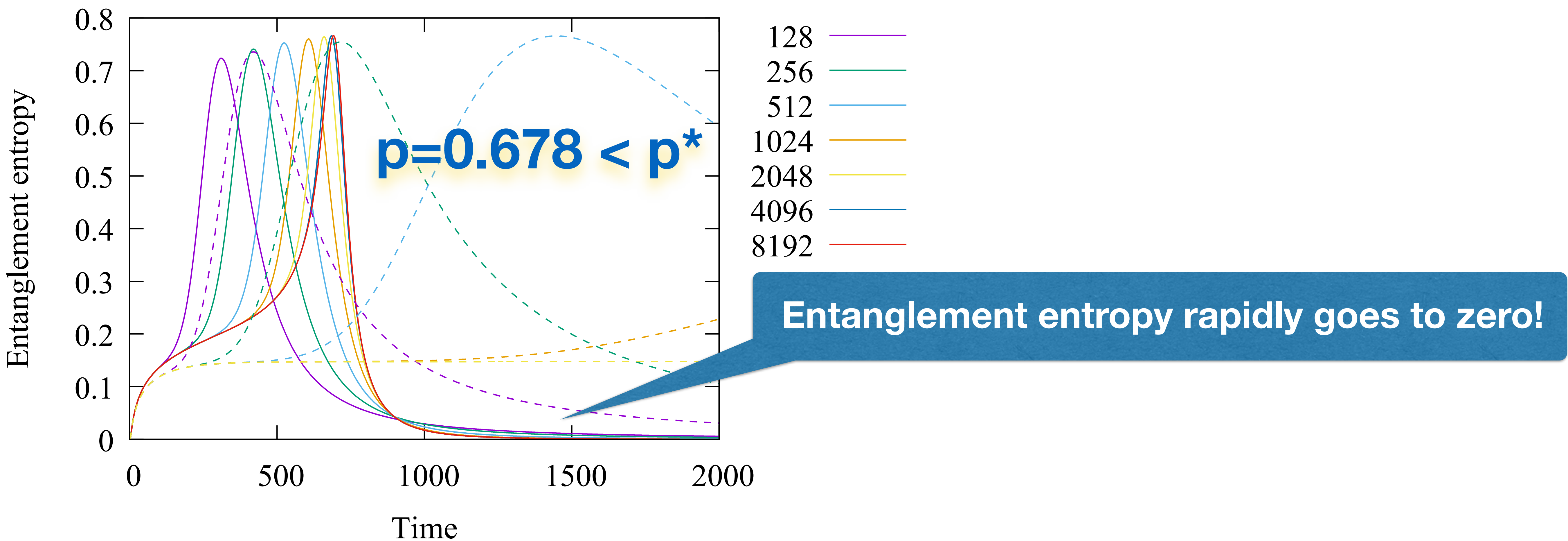
Harada & Kawashima, PRL **123**, 090601(2019).

Cusp at $p_2^*=0.6785(5)$



Discussion

Performance of MPS for DK



In the region p < p*, both of quasi-absorbing states and finite density states are important!