A new renormalization group on higher dimensional tensor networks

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TNSAA 2019-2020
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Dec. 6, 2019

Kadoh and Nakayama, arXiv:1912.02414

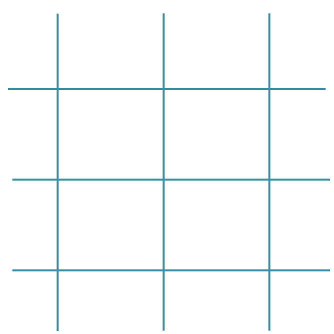
Motivation

- Models and methods with sign problem
  - QCD at finite density and theta vacuum
  - SUSY and chiral gauge theories
  - Hubbard model
  - real-time dynamics

    …

- Tensor Network method [Levin-Nave, 2007]

tensor network

$Z =$ 

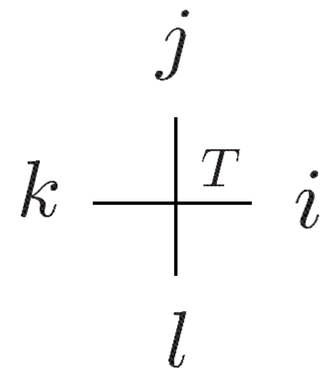
no stochastic process
& no sign problem

Contents

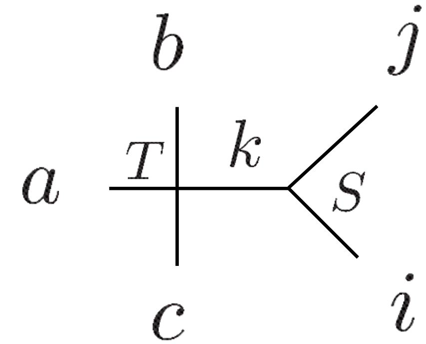
2. Tensor renormalization group

Graphical notation

$$T_{ijkl} =$$

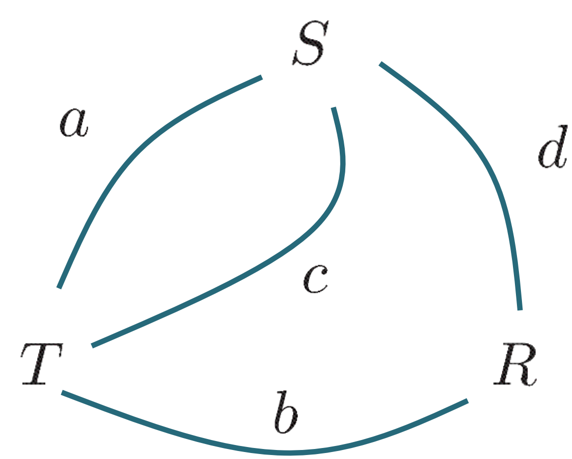


$$\sum_{k=1}^{N} T_{kabc} S_{ijk} =$$



“contraction”

$$Z = \sum_{a,b,c,d} T_{bca} S_{acd} R_{bd} =$$



“tensor network”

Singular value decomposition

- N x N matrix T_{IJ}

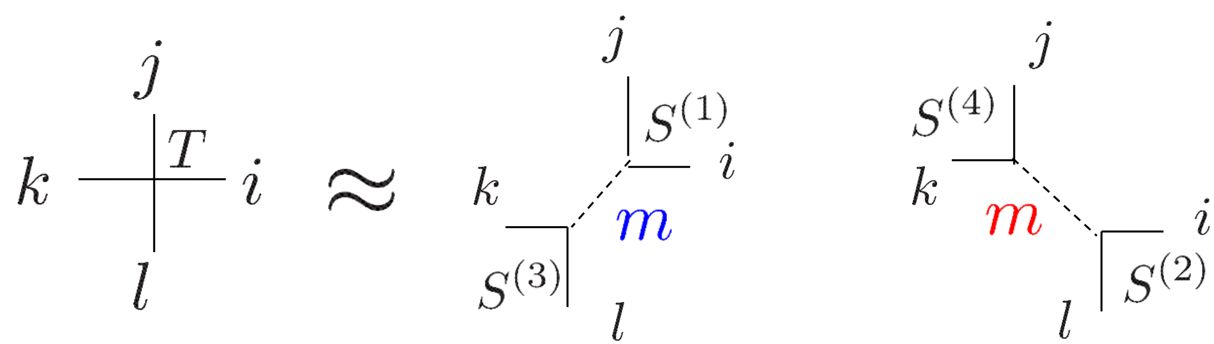
singluar values

$$T_{IJ} = \sum_{m=1}^{N} U_{Im} \sigma_m V_{mJ} \qquad \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_N \geq 0$$

$$\approx \sum_{m=1}^{D_{\rm cut}} S_{Im} S'_{mJ} \qquad D_{cut} < N$$

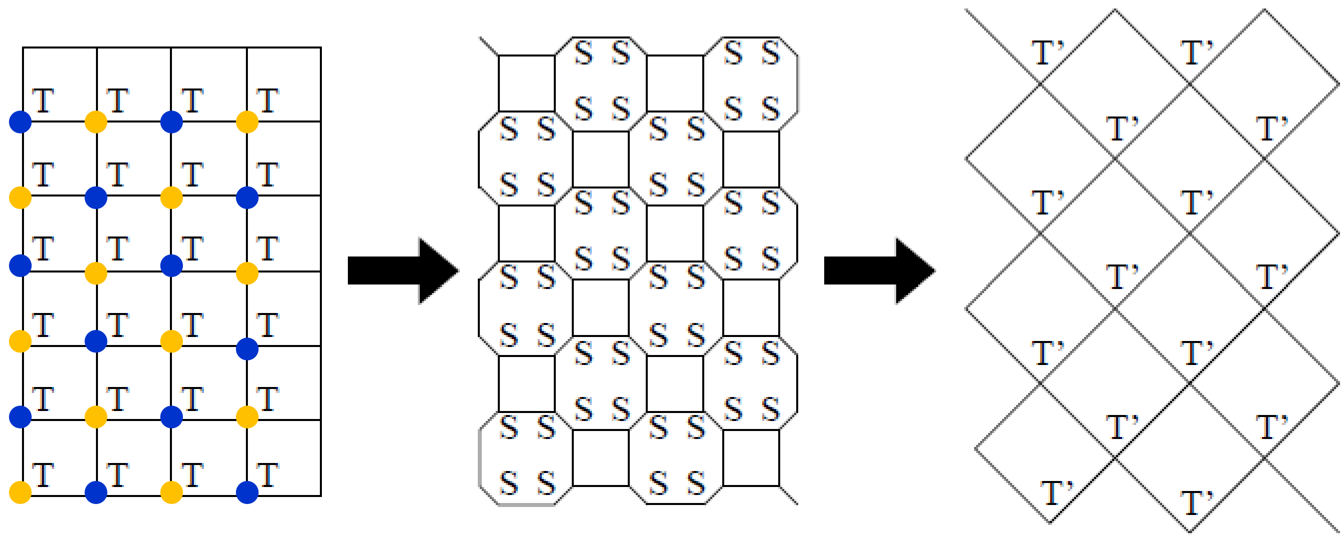
$$S_{Im} = \sqrt{\sigma_m} U_{Im}$$
$$S'_{mJ} = \sqrt{\sigma_m} V_{mJ}$$

- SVD of tensor



Tensor renormalization group

Levin-Nave, 2007



$$Z = \sum_{i,j,\dots} T_{ijkl} T_{lmno} \cdots \approx \sum_{i,j,\cdots} S^{(1)} S^{(2)} S^{(3)} S^{(4)} \dots = \sum_{m,n,\dots} T'_{mnpq} T'_{mkrs} \cdots$$

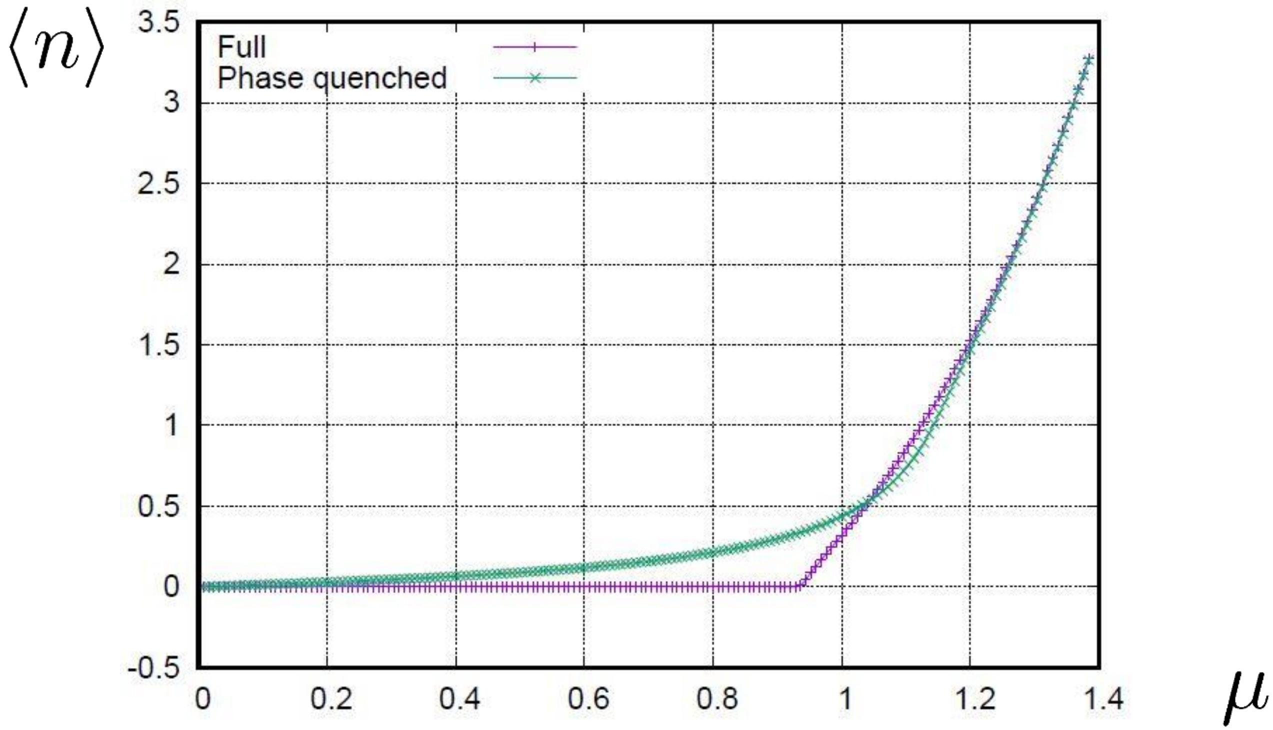
$$Z \approx \sum_{i,j=1}^{D_{\mathrm{cut}}} \tilde{T}_{ijij}$$

$$T'_{ijkl} = \sum_{abcd} S^{(1)}_{abi} S^{(2)}_{bcj} S^{(3)}_{cdk} S^{(4)}_{dal}$$

$$\#\ \text{of}\ T' = \frac{\#\ \text{of}\ T}{2}$$

2d complex ϕ^4 theory at finite density

DK, Kuramashi, Nakamura, Sakai, Takeda, Yoshimura
in preparation



Silver Blaze phenomena is clearly observed by tensor renormalization group.

3. Renormalization group on a triad network

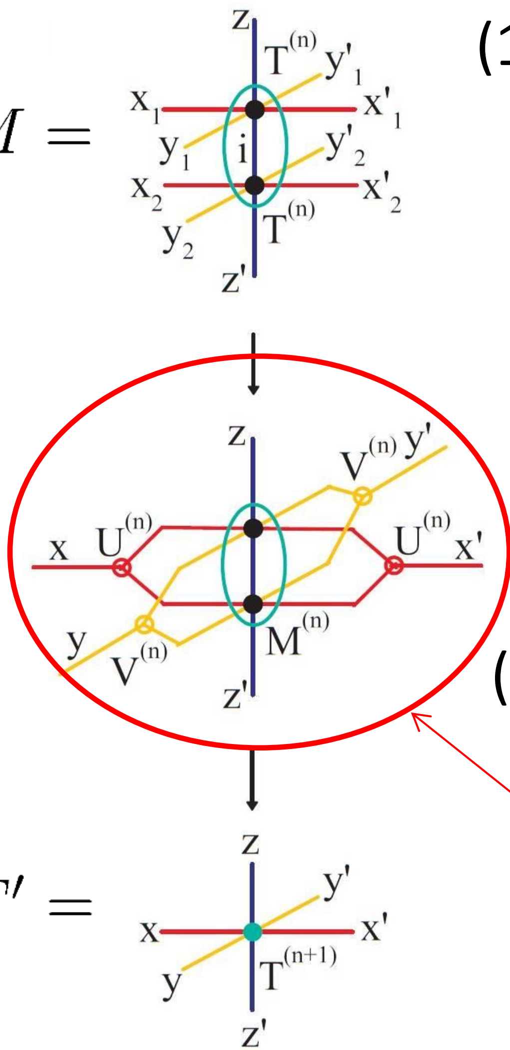
Kadoh and Nakayama, arXiv:1912.02414

Higher-order TRG (HOTRG)

Xie et al., 2012

3d case

$M =$



$T' =$

(1) Make projectors from two Ts

$$M = T \cdot T$$

$\mathcal{O}(D^{2d+2})$

diagonalization:

$$(UMM^{\dagger}U^{\dagger})_{XX'} = \sigma_X \delta_{XX'}$$

U_{X,x_1x_2} : projector for x-direction

(2) Take contractions with projectors

→ a renormalized tensor T'

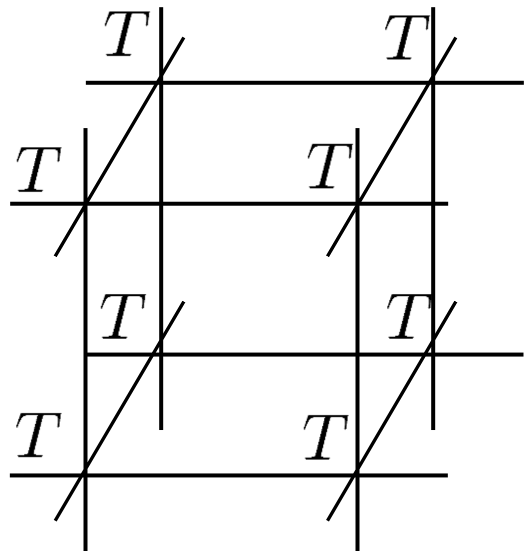
$\mathcal{O}(D^{4d-1})$

Can we wait?

Computation time for D=32 is a few hours for d=2.
However, for d=4, it becomes…

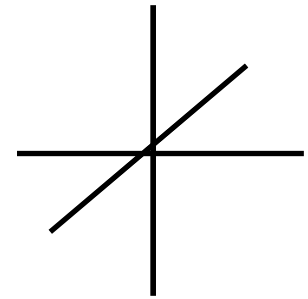
→ Need a low-cost renormalization scheme applicable to higher dimensions

Why the cost of HOTRG is high?



tensor networks on
hyper cubic lattice

$T =$

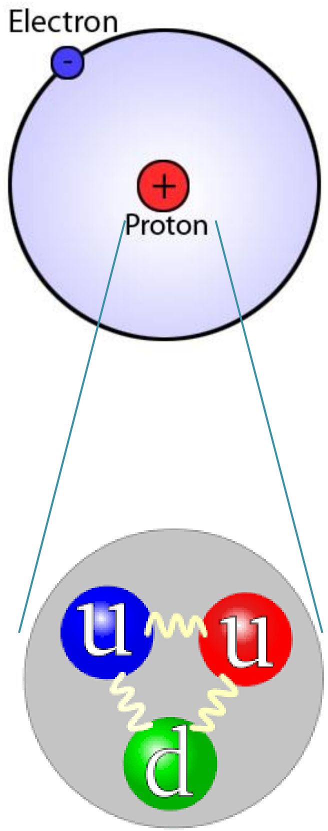


2d-rank tensor

The cost of contracting
two 2d-rank tensors
is high for large d.

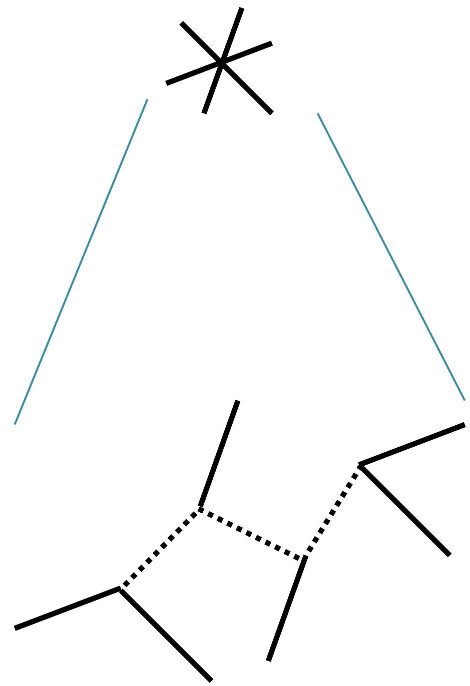
We have to reconsider a theory of tensor networks at a fundamental level.

Fundamental building blocks



quarks

rank-6 tensor



rank-3 tensors

“triads”

Hidden structure

A tensor is obtained as a CPD form for generic lattice models with nearest neighbor interaction:

$$T_{ijklmn} = \sum_{a=1}^{r} W^{(1)}_{ai} W^{(2)}_{aj} W^{(3)}_{ak} W^{(4)}_{al} W^{(5)}_{am} W^{(6)}_{an}$$

$$= \sum_{a,b,c=1}^{r} A_{ija} \, B_{akb} \, C_{blc} \, D_{cmn}$$

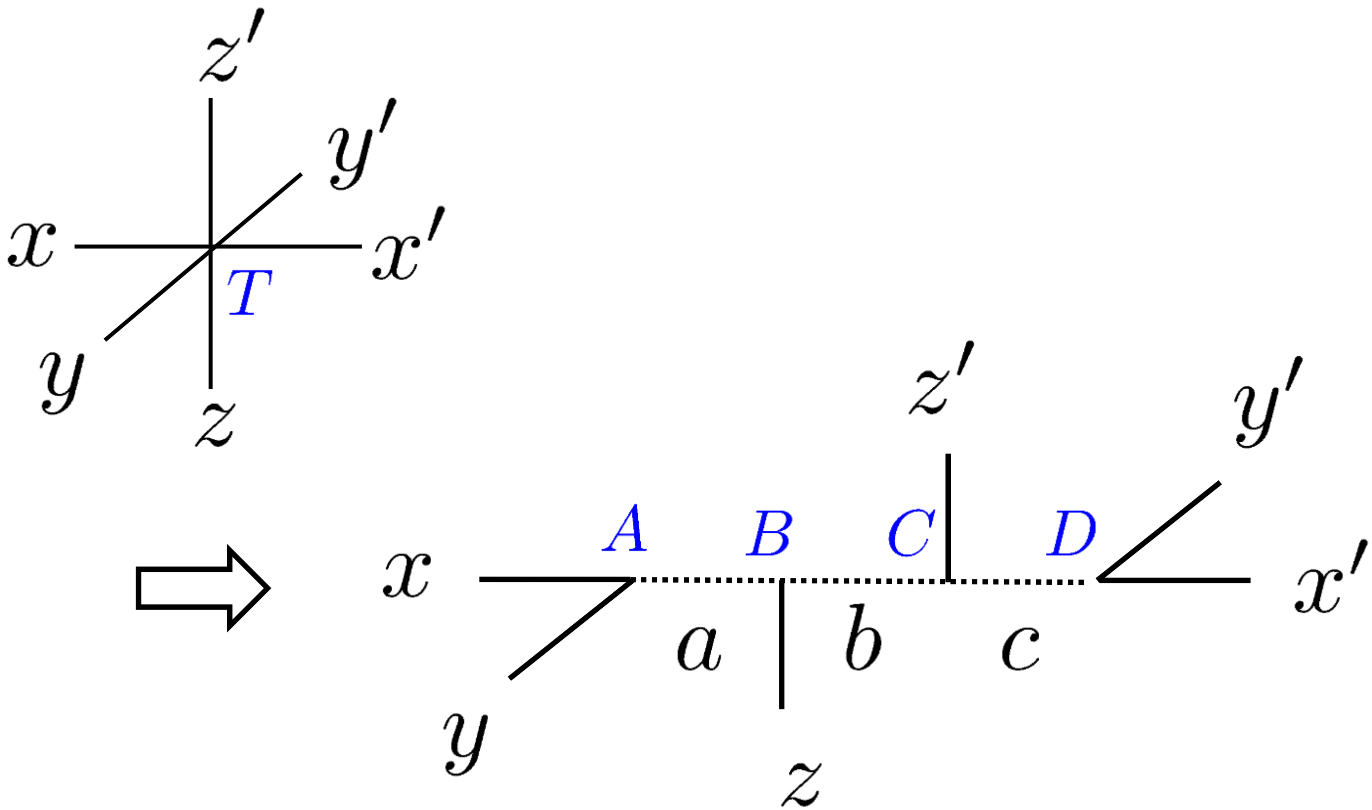
$$A_{ija} \equiv W^{(1)}_{ai} W^{(2)}_{aj} \qquad D_{cmn} \equiv W^{(5)}_{cm} W^{(6)}_{cn}$$

$$B_{akb} \equiv \delta_{ab} W^{(3)}_{ak} \qquad C_{blc} \equiv \delta_{bc} W^{(4)}_{bl}$$

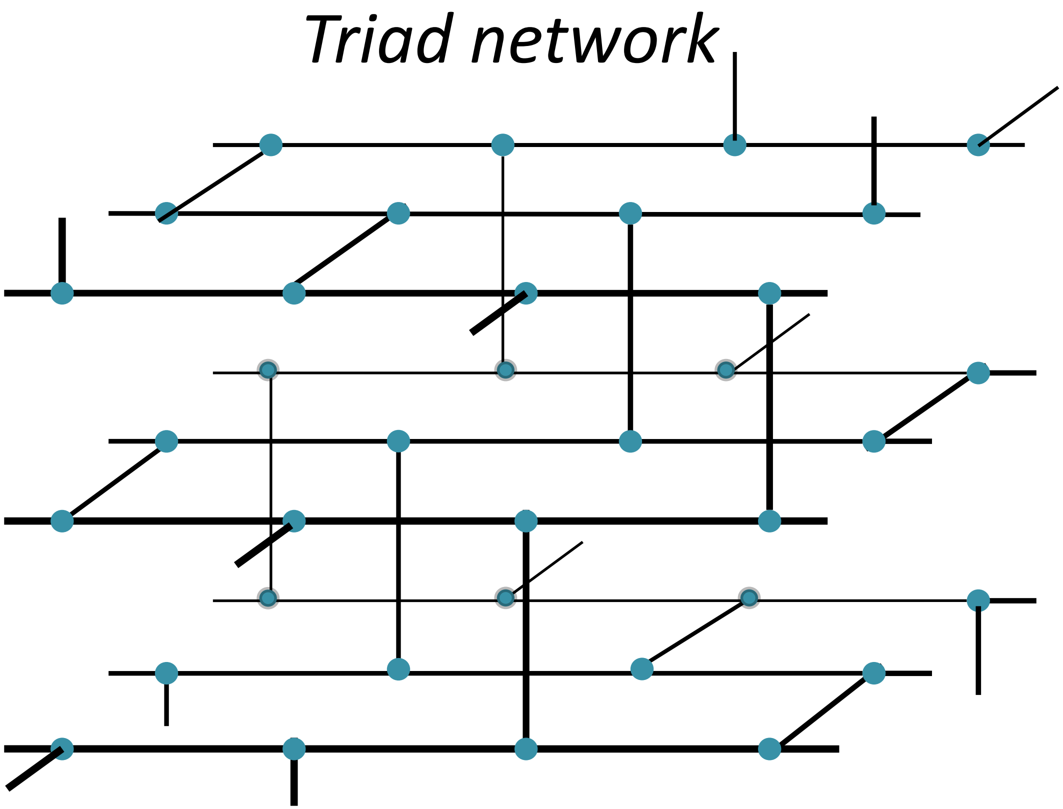
e.g.) 3d Ising model $W^{(\mu)} = W \equiv \begin{pmatrix} \sqrt{\cosh(\beta)} & \sqrt{\sinh(\beta)} \\ \sqrt{\cosh(\beta)} & -\sqrt{\sinh(\beta)} \end{pmatrix}.$

Triad representation of T

$$T_{xx'yy'zz'} = \sum_{a,b,c} A_{xya} \, B_{azb} \, C_{bz'c} \, D_{cy'x'}$$



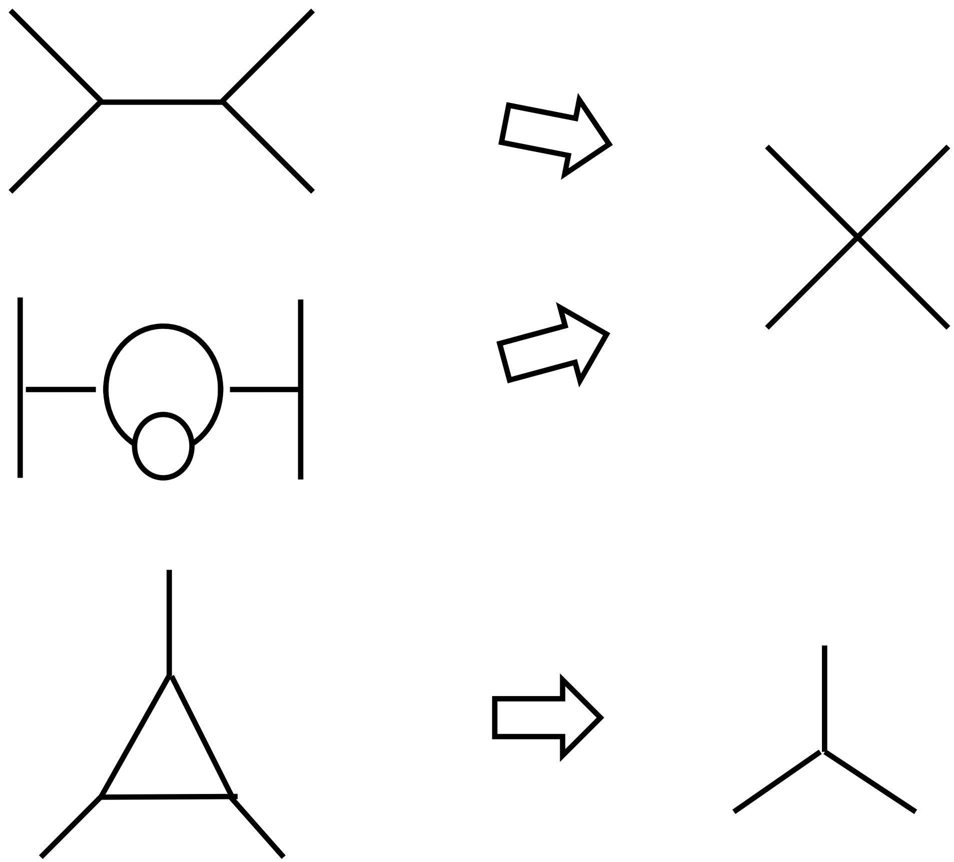
Triad networks and RGs



Cost of RGs on a triad network (Triad RGs) would be naturally reduced.

D^5-cost operations

graphs with rank-3 tensors



rank-4 tensor

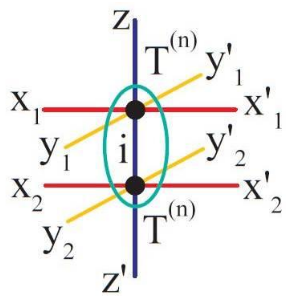
rank-3 tensor

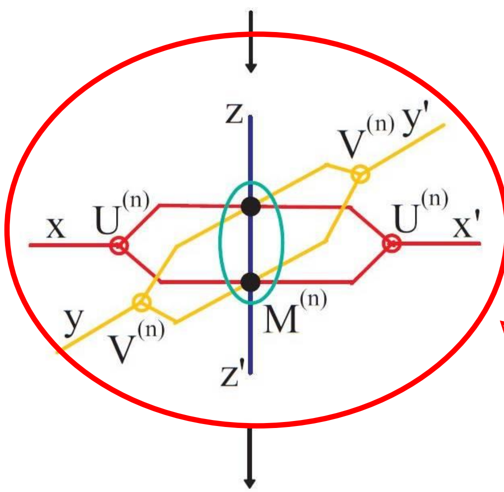
Higher-order TRG (HOTRG)

Xie et al., 2012

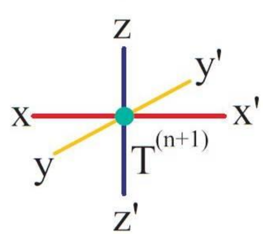
3d case

$M =$





$T' =$



$$M = T \cdot T$$

$$(UMM^\dagger U^\dagger)_{XX'} = \sigma_X \delta_{XX'}$$

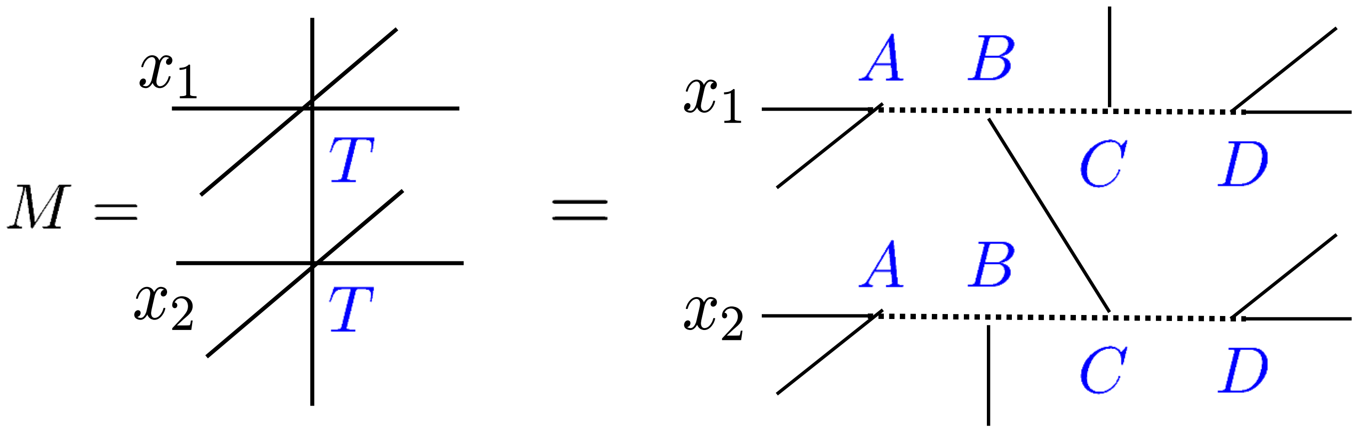
$U_{X,x_1 x_2}$: projector

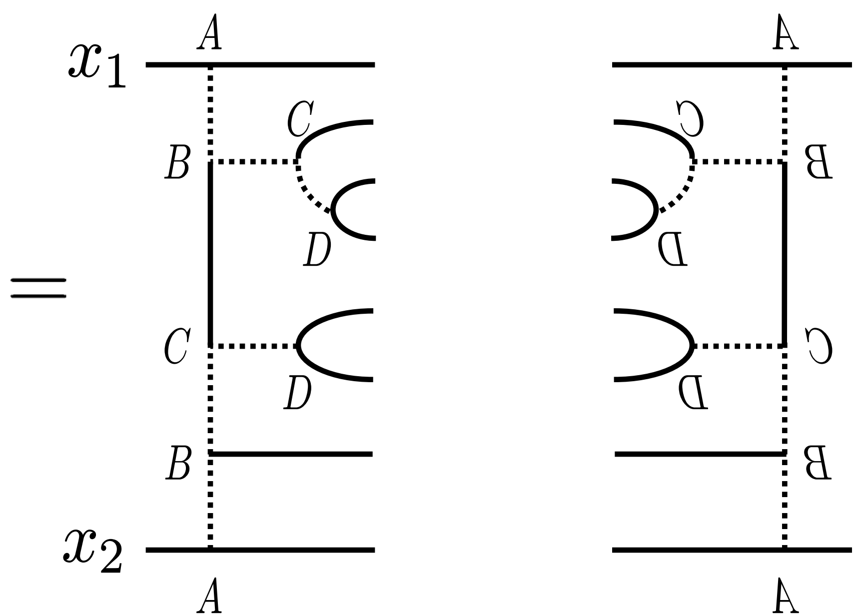
$\mathcal{O}(D^{2d+2})$

Contractions for making a renormalized tensor

$\mathcal{O}(D^{4d-1})$

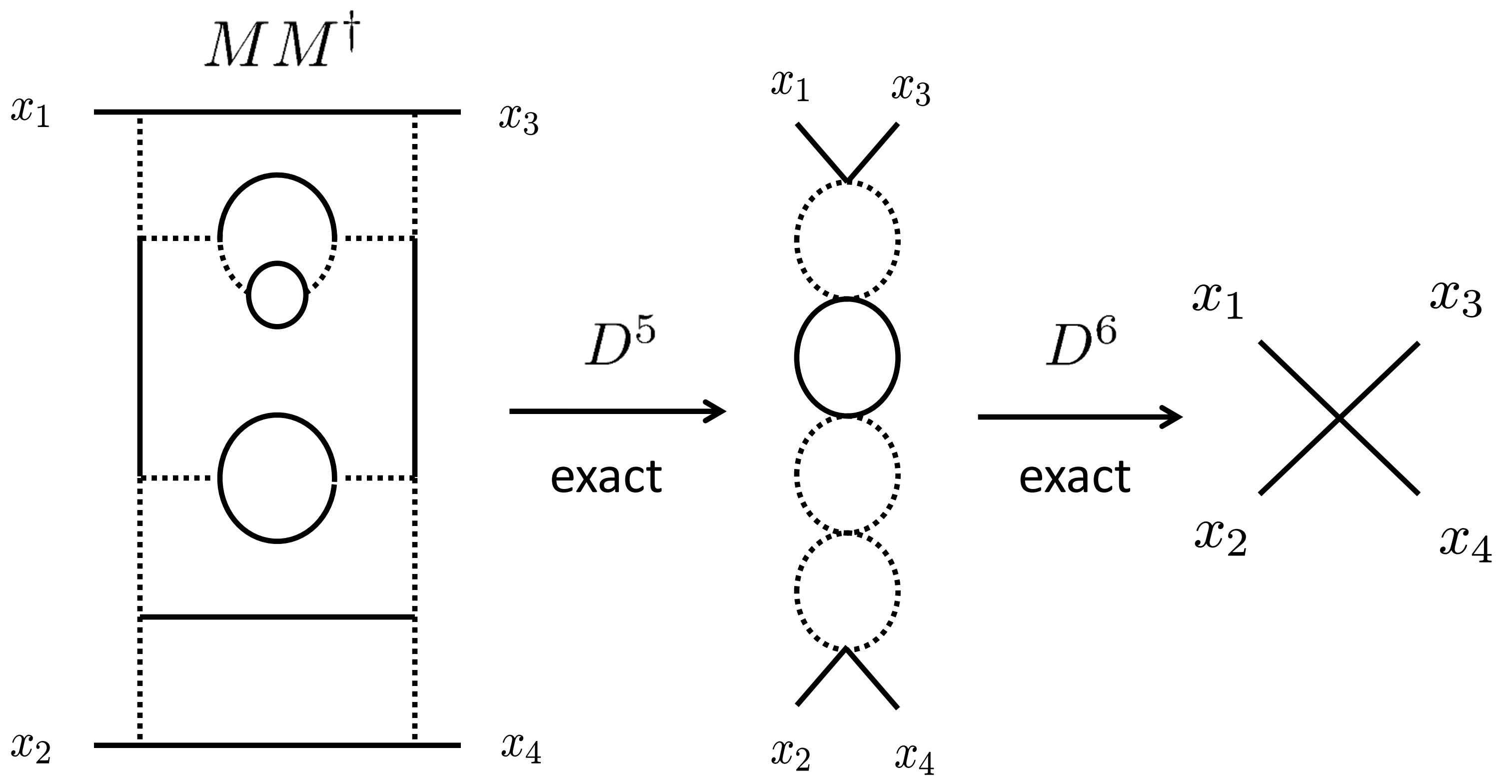
M in the triad representation





$M^\dagger$ is a mirror image of M.

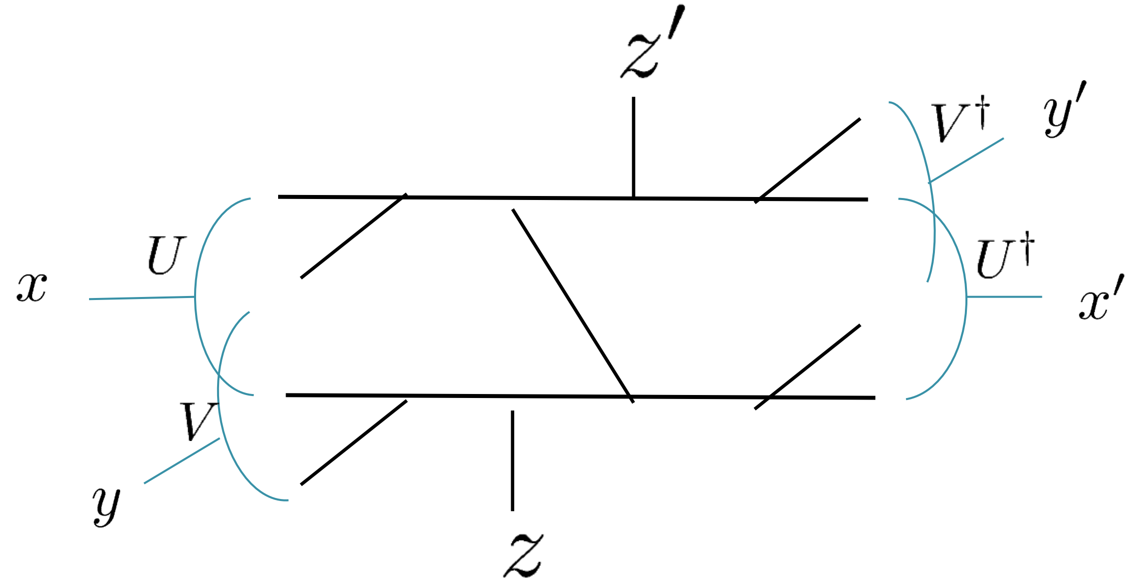
Steps of making projectors



Projectors U can be exactly prepared at an $\mathcal{O}(D^6)$ cost in any dimension!

→ $\mathcal{O}(D^5)$ by using a randomized SVD

Contraction of two triads



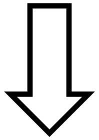
2 projectors

U_{X,x_1x_2}

V_{Y,y_1y_2}

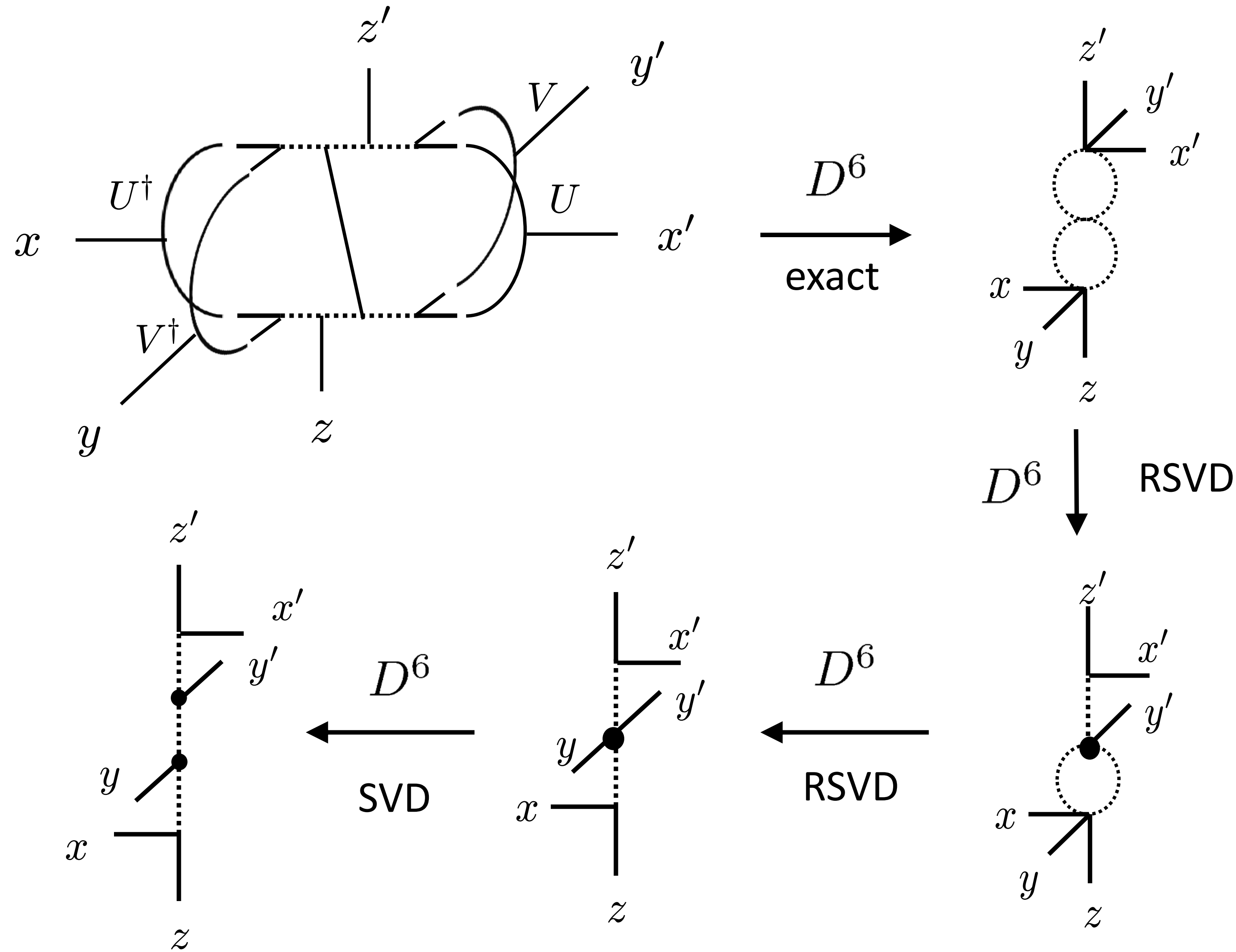
Main cost comes from contractions & RSVDs:

$\mathcal{O}(D^{d+3})$



a renormalized triad

Steps of making a renormalized triad

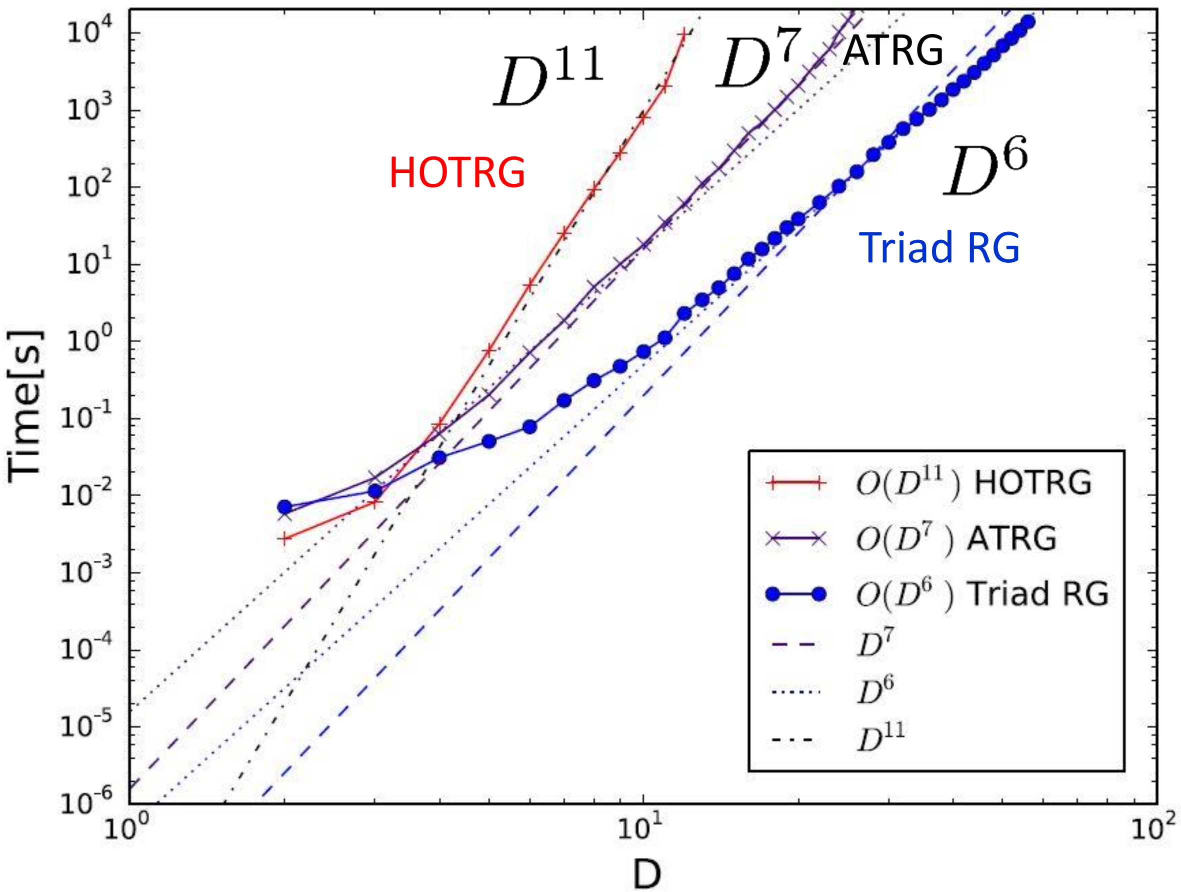


Theoretical cost

	dimensionality			
	2	3	4	d
TRG	D^5	—	—	—
HOTRG	D^7	D^{11}	D^{15}	D^{4d-1}
Anisotropic TRG [Adachi et al.,2019]	D^5	D^7	D^9	D^{2d+1}
Triad RG (this work)	D^5	D^6	D^7	D^{d+3}

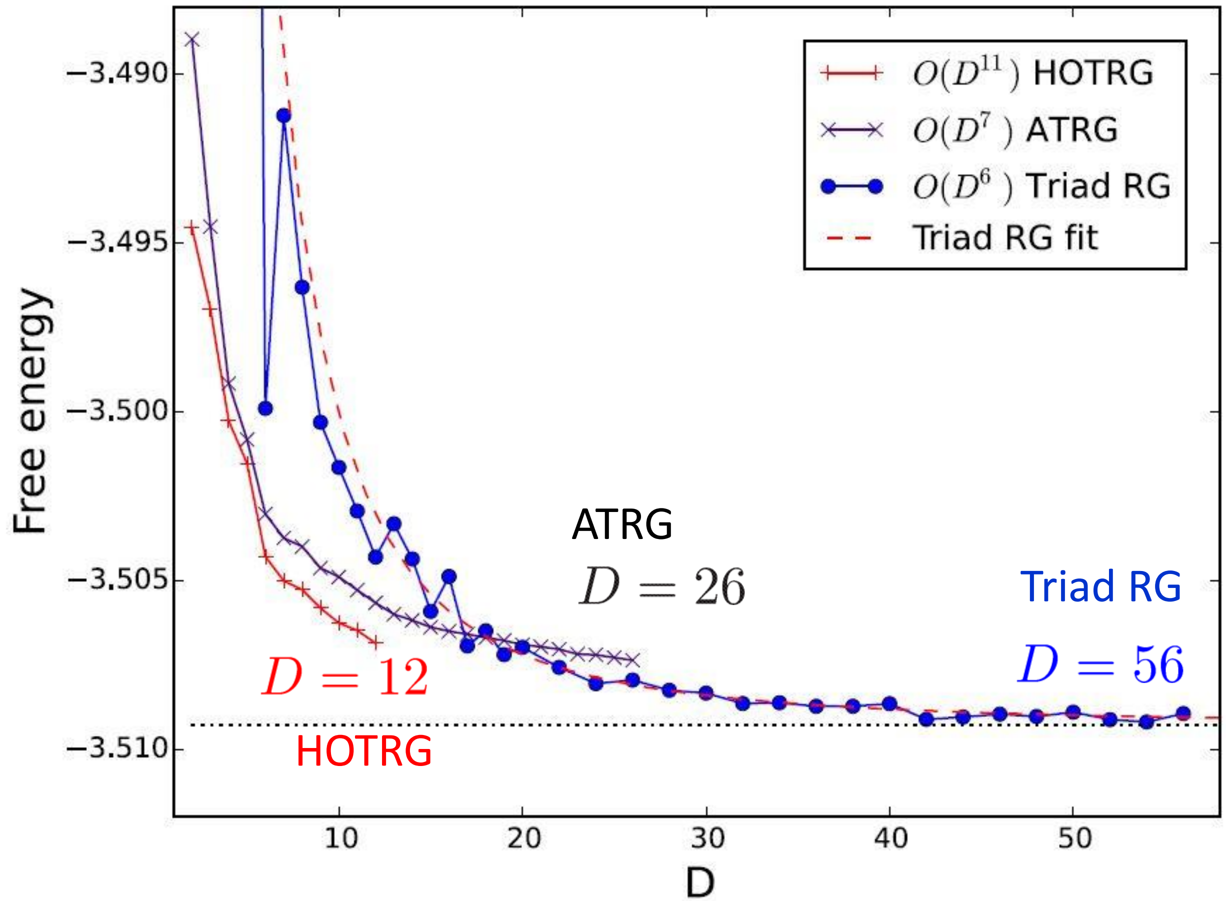
Numerical test in 3d Ising model at Tc

Computational time vs. D



Theoretical D-dependence is properly reproduced in actual computations.

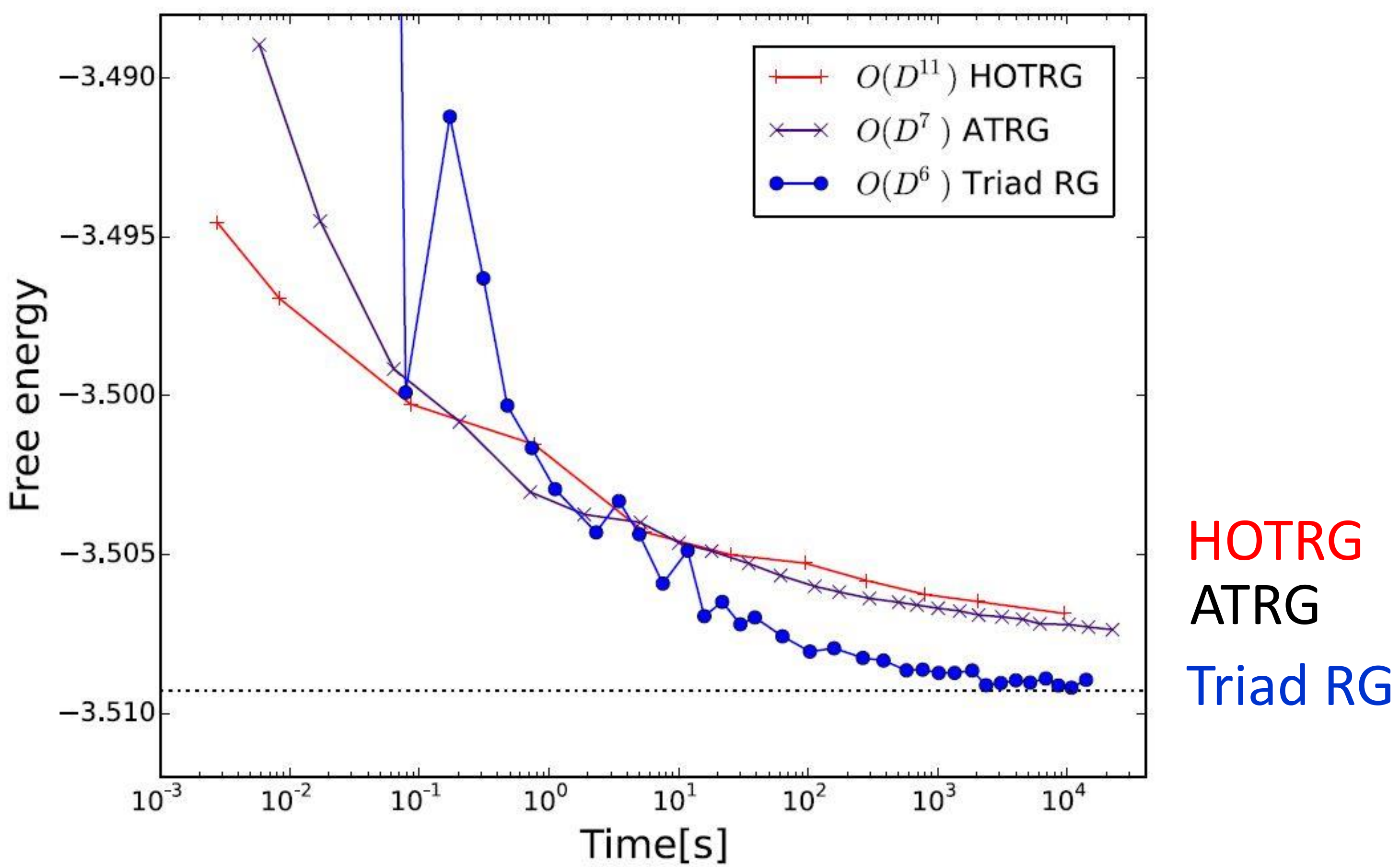
D-dependence of free energy



a few hours using laptop computers

The Triad RG method shows good convergence as D increases.

Free energy vs. computational time



The other methods need much more time to approach a converged value around -3.509.

4. Future outlook

Triad RGs and its application

different types of Triad RG

fermionic systems such as Hubbard model

higher dimensional QFTs

improvements for Triad RGs

gauge theory

Triad RGs change the time …

Triad RGs change the time ...

3 days fun time

Thank you