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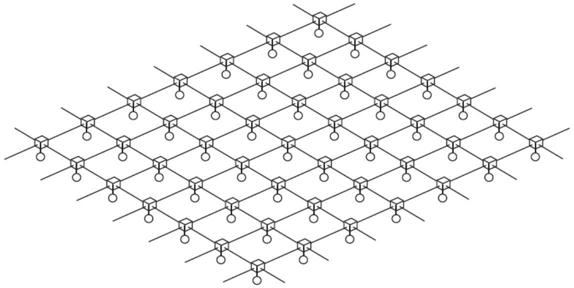
CBSM²

Understanding Kitaev-Related Models through Tensor Networks

2019.12.04 Naoki KAWASHIMA (ISSP)

東京大学
物性研究所

Tensor Network (TN)



(1) Classical statistical mechanical models are TNs
--- corner transfer matrix (Baxter, Nishino, Okunishi, ...)

(2) TN is the "right" representation for renormalization group
--- "scale-invariant tensor", various techniques (TRG, TNR, loop-TNR, MERA, HOTRG, ...) (Gu, Levin, Wen, Vidal, Evenbly, Xiang, ...)

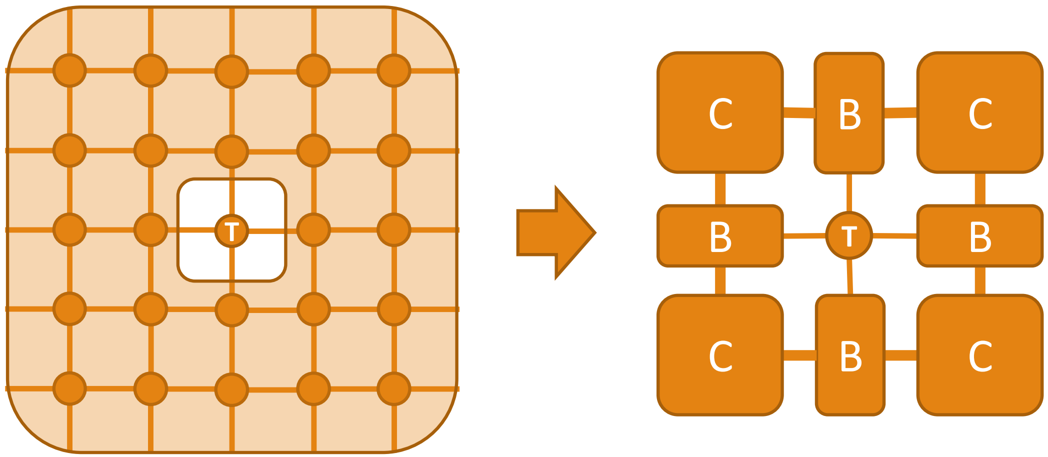
(3) TN is the "right" language for expressing topological/gauge structure
--- projective representation, symmetry-protected topological phases (Chen, Gu, Wen, Verstraete, Cirac, Schuch, Perez-Garcia, Oshikawa, ...)

(4) TN connects physics to informatics
--- application of/to data science, machine learning, automatic differentiation, lattice QCD, AdS/CFT-correspondence, etc.

Import from Stat. Mech. to TN

Example: Corner transfer matrix (CTM)

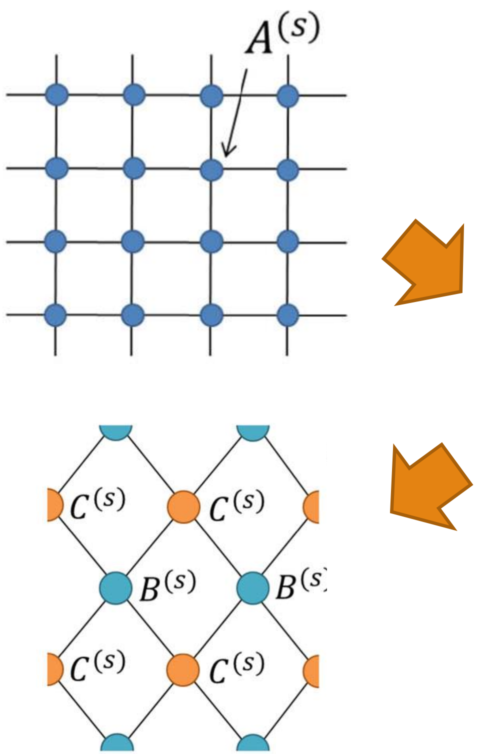
Baxter: J. Math. Phys 9, 650 (1968); J. Stat. Phys. 19, 461 (1978)
Nishino, Okunishi: JPSJ **65**, 891 (1996)
R. Orus *et al*: Phys. Rev. B **80**, 094403 (2009)

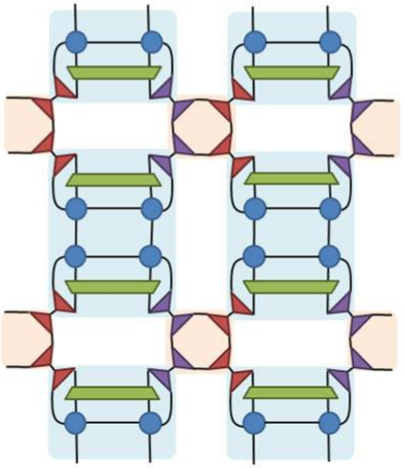


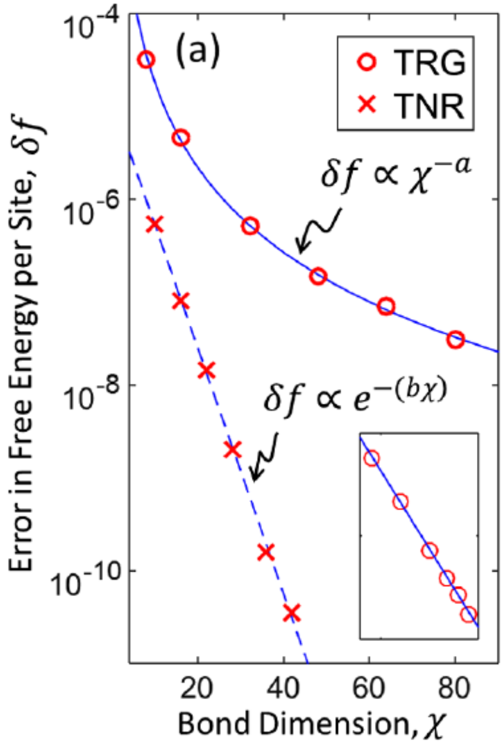
Effect of the infinite environment is approximated by B and C, which are obtained by iteration/self-consistency.

TN-based real-space RG

Example: TNR (Evenbly-Vidal 2015)



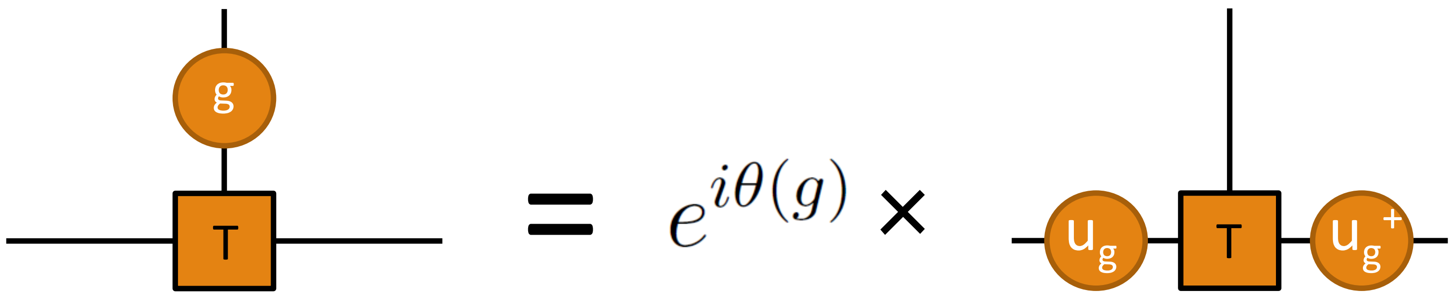




Now, RSRG is not just an idea or a crude approximation,
but a **systematic and precise** method.

Topological/gauge structure in TN

Example: Characterization of SPT phase Chen,Gu,Liu,Wen (2012)



$$u_g u_h = \boxed{\omega(g,h)} u_{gh}$$

For the symmetry group G, an SPT phase is characterized by the 2nd cohomology group H2(G,U(1)) of the projective representation of G.

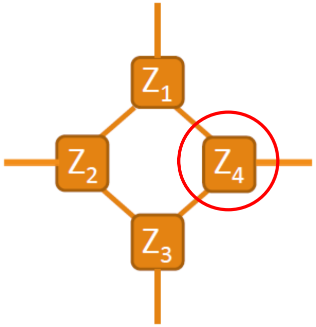
TN for data science

Example: Image classification by TN

Zhao, Cichocki et al, arXiv:1606.05535

COIL100 2D image classification task
128 x 128 x 3 x 7200 bits

Decomposed the whole data into a tensor ring, and applied KNN classifier (K=1) to the image-identifying core (Z_4).

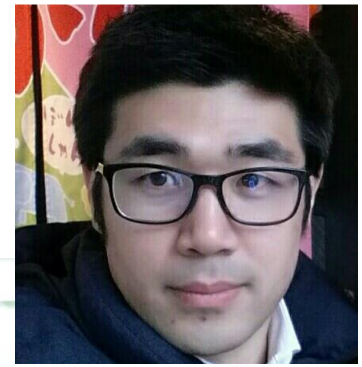


Ring decomposition shows better performance than open chain (TT).

We may use TN for identification of subtle characters.

Collaborators

Hyun-Yong LEE (ISSP)
Tsuyoshi OKUBO (U. Tokyo)
Ryui KANEKO (ISSP)
Yohei YAMAJI (U. Tokyo)
Yong-Baek Kim (Toronto)

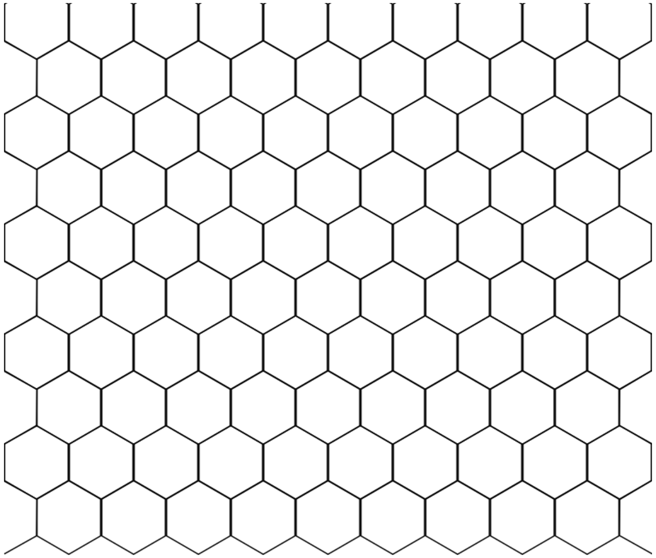


Kitaev Model

Kitaev, Ann. Phys. 321 (2006) 2

$$H = \sum_{(ij)} \sum_{\mu=x,y,z} J^{\mu}_{ij} \sigma^{\mu}_{i} \sigma^{\mu}_{j}$$

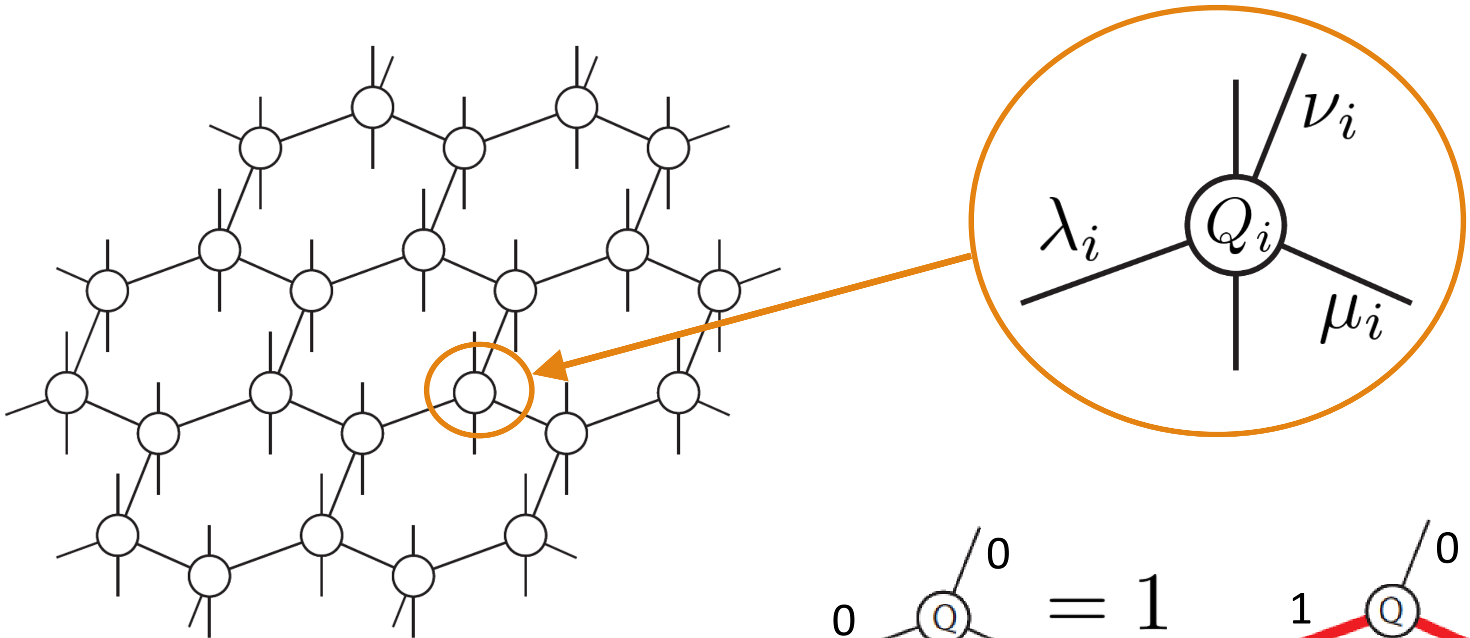
$$J^{\mu}_{ij} = \begin{cases} J & ((ij) \parallel \mu\text{-axis}) \\ 0 & (\text{otherwise}) \end{cases}$$

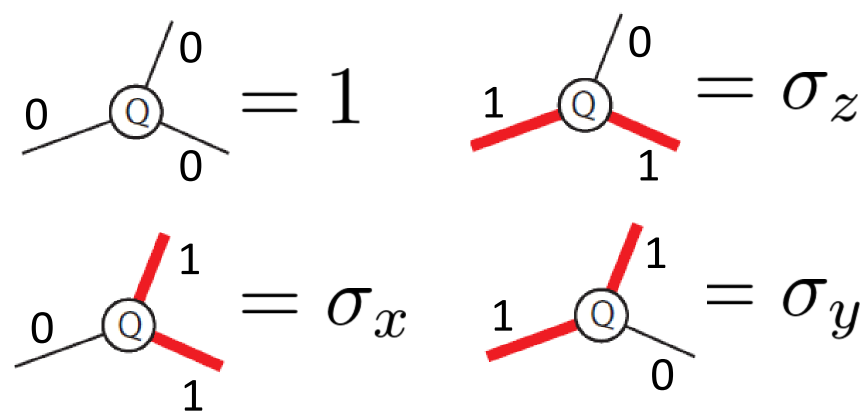


(1) Symmetries (rotation, translation, t-rev.)
(2) Flux-free
(3) Gapless (2D Ising Universality Class)

Lee, Kaneko, Okubo, NK: PRL 123, 087203 (2019)

Loop Gas Projector (LGP)





$$Q_{\mathrm{LG}} \equiv \sum_{\{\lambda_i\},\{\mu_i\},\{\nu_i\}} Q_i^{\lambda_i \mu_i \nu_i}$$

$$\lambda_i, \mu_i, \nu_i = 0, 1$$

Lee, Kaneko, Okubo, NK: PRL 123, 087203 (2019)

LGP is projector to flux-free space

$$P_{\mathrm{LG}} \equiv \frac{1}{|\Gamma|} \sum_{\{\lambda_i\},\{\mu_i\},\{\nu_i\}} \prod_i Q_i^{\lambda_i \mu_i \nu_i} = \frac{1}{|\Gamma|} \sum_{G \in \Gamma} Q(G)$$

$$Q(G) = \prod_i Q_i^{\lambda_i(G)\mu_i(G)\nu_i(G)}$$

$$P_{\mathrm{LG}}^2 = P_{\mathrm{LG}} \qquad W_p P_{\mathrm{LG}} = P_{\mathrm{LG}} W_p = P_{\mathrm{LG}}$$

Loop Gas State $\quad |\psi_0\rangle \equiv |\mathrm{LGS}\rangle \equiv P_{\mathrm{LG}} |(111)\rangle$

LGS is flux-free and non-magnetic

Lee, Kaneko, Okubo, NK: PRL 123, 087203 (2019)

Loop Gas State

Lee, Kaneko, Okubo, and NK, to appear in PRL (2019)

$$|\mathrm{LGS}\rangle = \sum_{G:\,\text{loop config.}} Q(G)|(111)\rangle$$

$|(111)\rangle$ ... fully-polarized state in (111) direction

$=$

$+\cdots$

$$\langle \mathrm{LGS}|\mathrm{LGS}\rangle = Z_{\mathrm{LG}}\left(\zeta = \frac{1}{\sqrt{3}}\right) = Z_{\text{Trianglular Ising}}(T = T_c)$$

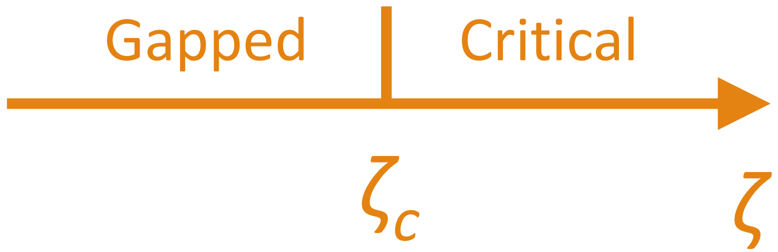
Exactly at the critical point.

Lee, Kaneko, Okubo, NK: PRL 123, 087203 (2019)

Classical Loop Gas

B. Nienhuis, Physical Review Letters 49, 1062 (1982).

$$Z_{\mathrm{LG}}(n,\zeta) \equiv \sum_{G:\,\text{loop config.}} n^{N_{\mathrm{loop}}(G)} \zeta^{|G|}$$



$$\zeta_c(n) = \frac{1}{\sqrt{2+\sqrt{2-n}}}$$

$$\zeta_c(1) = \frac{1}{\sqrt{3}}$$

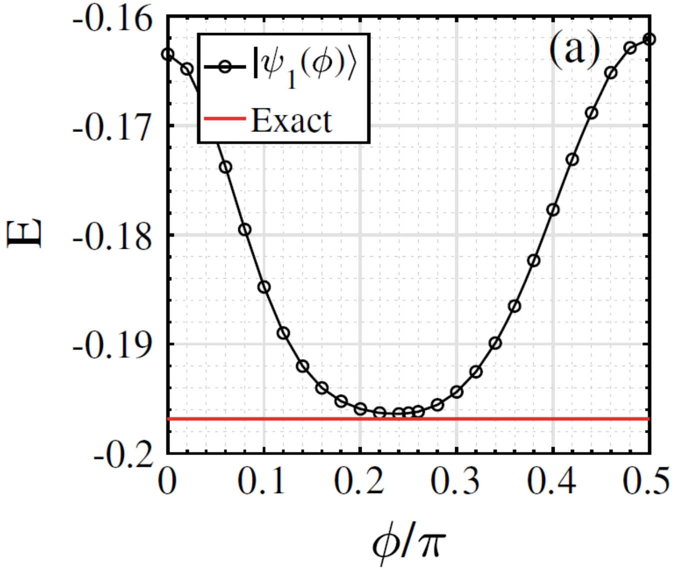
LGS is gapless and belongs to the 2D Ising universality class (the same as the KHM ground st.)

Lee, Kaneko, Okubo, NK: PRL 123, 087203 (2019)

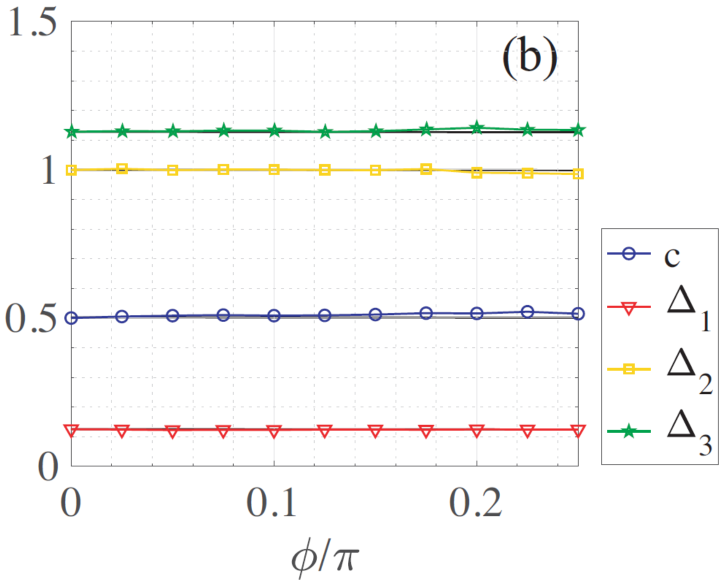
Better variational function ψ_1

$$\psi_1 \equiv R_{DG}(\phi)|\mathrm{LGS}\rangle$$

$R_{DG}(\phi)$ is analogous to P_{LG} with loops replaced by dimers.
(tan ϕ is the fugacity of the dimers)



loop-TNR + CFT

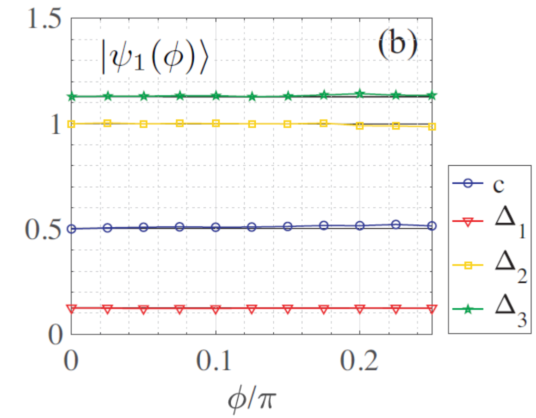


2D Ising universality class for any φ

Lee, Kaneko, Okubo, NK: PRL 123, 087203 (2019)

Series of Ansatzes

$$|\psi_0\rangle \equiv P_{\mathrm{LG}}|(111)\rangle = |\mathrm{LGS}\rangle$$
$$|\psi_1\rangle \equiv P_{\mathrm{LG}} R_{\mathrm{DG}}(\phi_1)|(111)\rangle$$
$$|\psi_2\rangle \equiv P_{\mathrm{LG}} R_{\mathrm{DG}}(\phi_1) R_{\mathrm{DG}}(\phi_2)|(111)\rangle$$

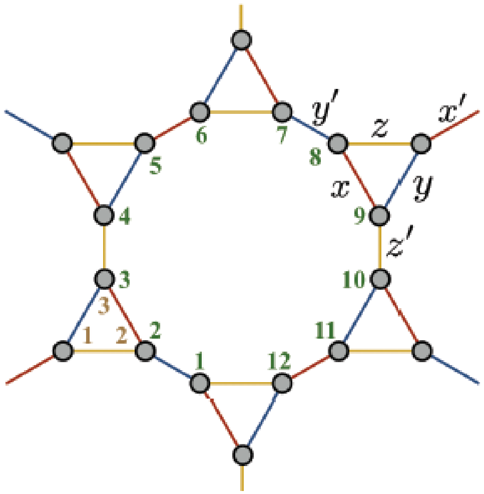


	ψ0=LGS	ψ1	ψ2	KHM gr. st.
# of prmtrs.	0	1	2	
E/J	-0.16349	-0.19643	-0.19681	-0.19682
ΔE/E	0.17	0.02	0.00007	-

Best accuracy by numerical calculation is achieved with only two tunable parameters

Kitaev Model on Star Lattice

$$\hat{\mathcal{H}} = -\frac{J}{4}\sum_{\langle ij\rangle\in\gamma}\hat{\sigma}_i^{\gamma}\hat{\sigma}_j^{\gamma} - \frac{J'}{4}\sum_{\langle ij\rangle\in\gamma'}\hat{\sigma}_i^{\gamma'}\hat{\sigma}_j^{\gamma'}$$



The ground state is CSL

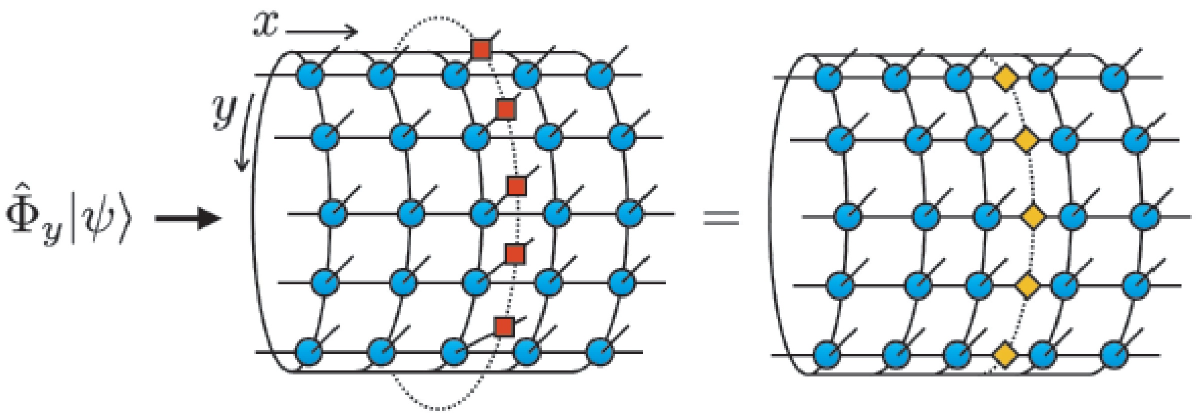
H. Yao and S. A. Kivelson, PRL99, 247203 (2007)
S. B. Chung, et al, PRB 81, 060403 (2010)

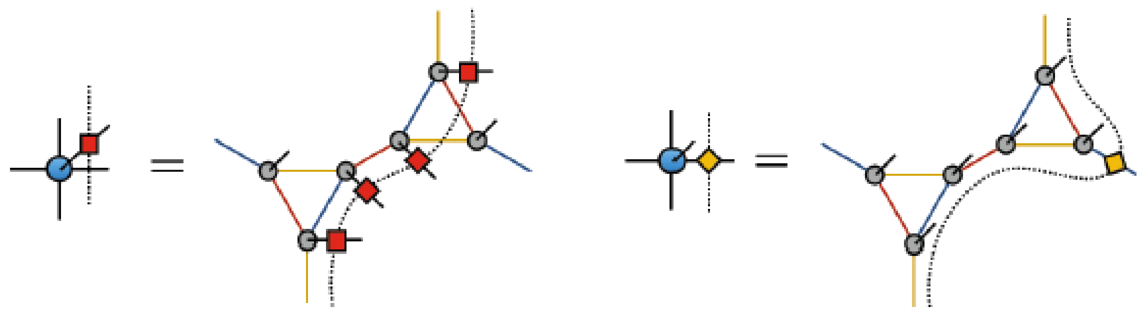
On the torus,

Abelian CSL ... 4-fold degenerate

non-Abelian CSL ... 3-fold degenerate

Effect of Global-Flux Operator





Global flux op. is equivalent to the global gauge twist.

Minimally Entangled States (MES)

Zhang, Grover, Turner, Oshikawa, and Vishwanath, PRB85, 235151 (2012)

For $|\psi\rangle = |\mathrm{LG}\rangle$ or $|\mathrm{SG}\rangle$,

$$|I\rangle = |\psi\rangle + \hat{\Phi}_y |\psi\rangle \quad \text{(trivial)}$$

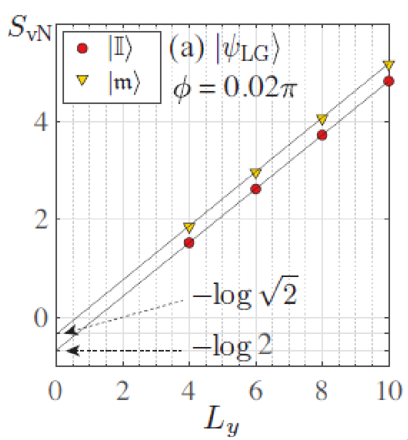
$$|m\rangle = |\psi\rangle - \hat{\Phi}_y |\psi\rangle \quad \text{(vortex)}$$

Lee, Kaneko, Okubo and NK: 1907.02268

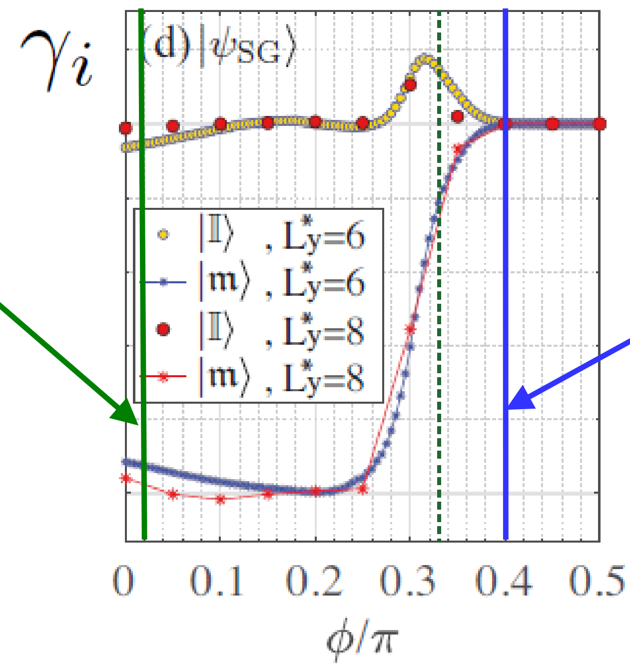
Topological Entanglement Entropy

$$S = \alpha L_y - \gamma_i$$

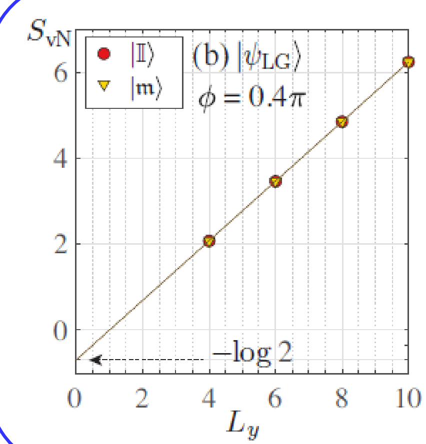
non-Abelian



TEE depends on
the top. excitation
(Ising anyon)



Abelian



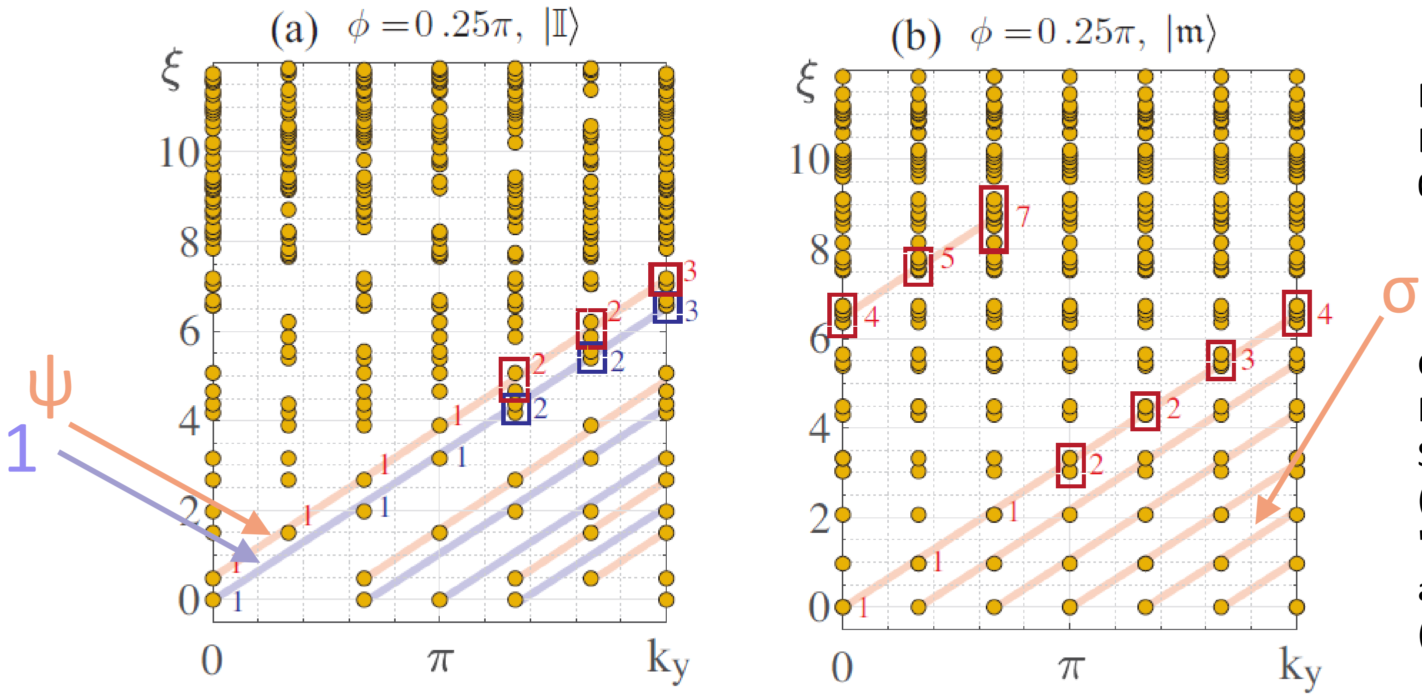
TEE does not depends
on the top. excitation
(the same as toric code)

C. Nayak, et al, RMP80, 1083 (2008)
A. Kitaev, Ann. Phys. 321, 2 (2006)

Lee, Kaneko, Okubo and NK: 1907.02268

Conformal Information of Chiral Edge Modes

$L_y = 6$ at $\phi = 0.25\pi$ (in non-Abelian phase)



Entanglement Spectrum: Li and Haldane, PRL 101, 010504 (2008)

Conformal Tower: Friedan, Qiu, and Shenker, PRL52, 1575 (1984), Henkel, et al, "Conformal Invariance and Critical Phenomena" (Springer, 1999)

Clear evidence for the chiral edge mode characterized by the Ising universality class

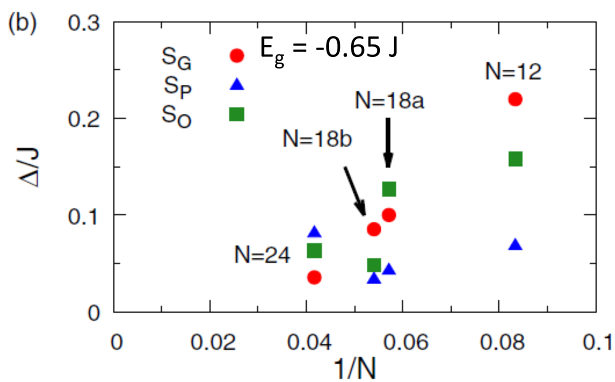
S=1 Kitaev Model

Baskaran, Sen, Shanker: PRB78, 115116 (2008)

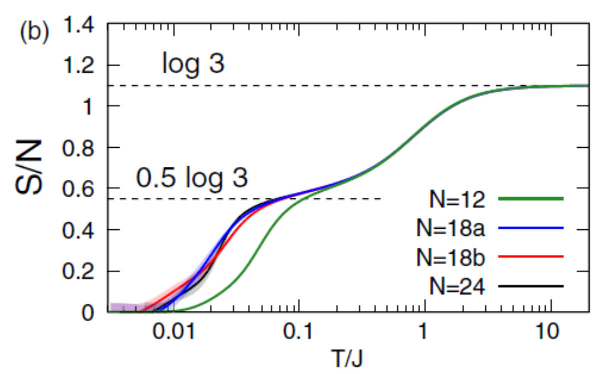
$$W_p \equiv e^{i\pi(S_1^y+S_2^z+S_3^x+S_4^y+S_5^z+S_6^x)}$$ (any S) $[H,W_p]=0$

The ground state is non-magnetic.

Koga, Tomishige, Nasu: JPSJ87, 063703 (2018) Exact diagonalization N<=24



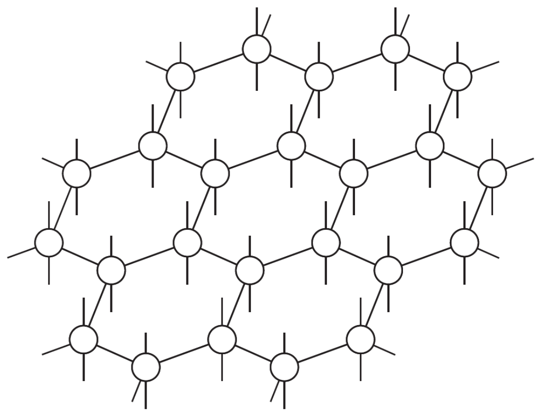
gapless?

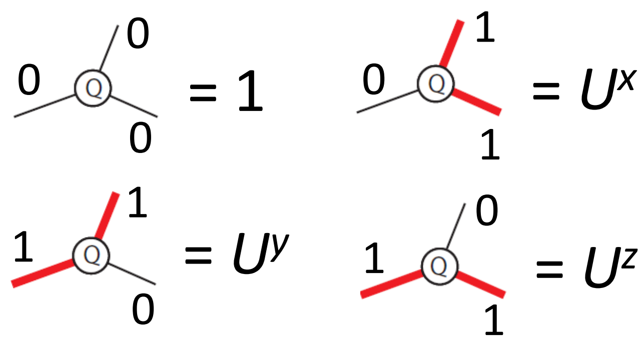


2-step structure analogous to S=1/2

Lee, NK, Kim: arXiv:1911.07714

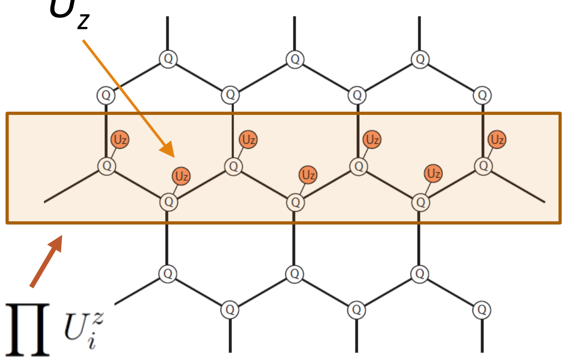
Loop-gas projector for S=1

$P_{LG} =$ 



σ replaced by π-rotations

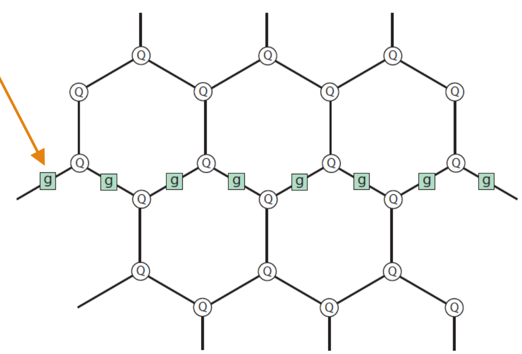
$$U^{\gamma} = e^{i\pi S^{\gamma}}$$



$$W_z \equiv \prod_{i \in \Gamma_z} U_i^z$$

the global flux operator

=



$$g = \begin{cases} \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} & \left(S = \frac{1}{2}\right) \\ \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} & (S = 1) \end{cases}$$

A LG projected state is always an eigenstate of the global flux operator.

S=1 loop-gas state

Fugacity of the loop gas is

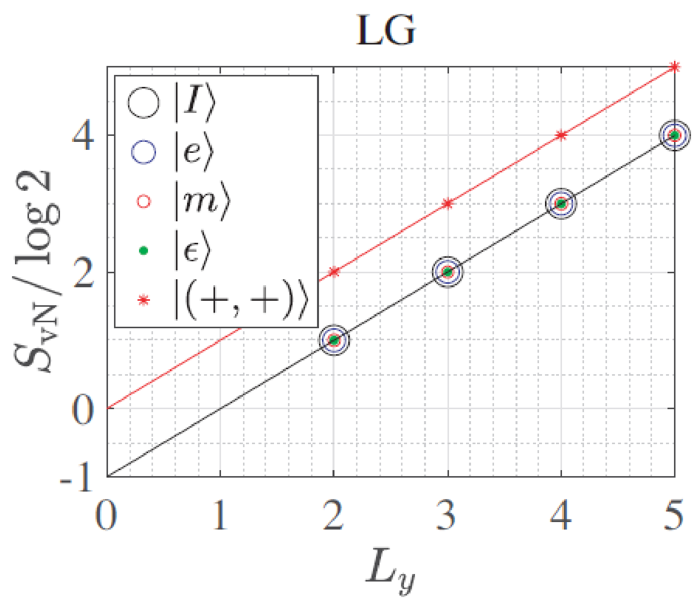
$$\langle (111) | U^{\gamma} | (111) \rangle = \begin{cases} \frac{1}{\sqrt{3}} & (S = \frac{1}{2}) \\ \frac{1}{3} & (S = 1) \end{cases}$$

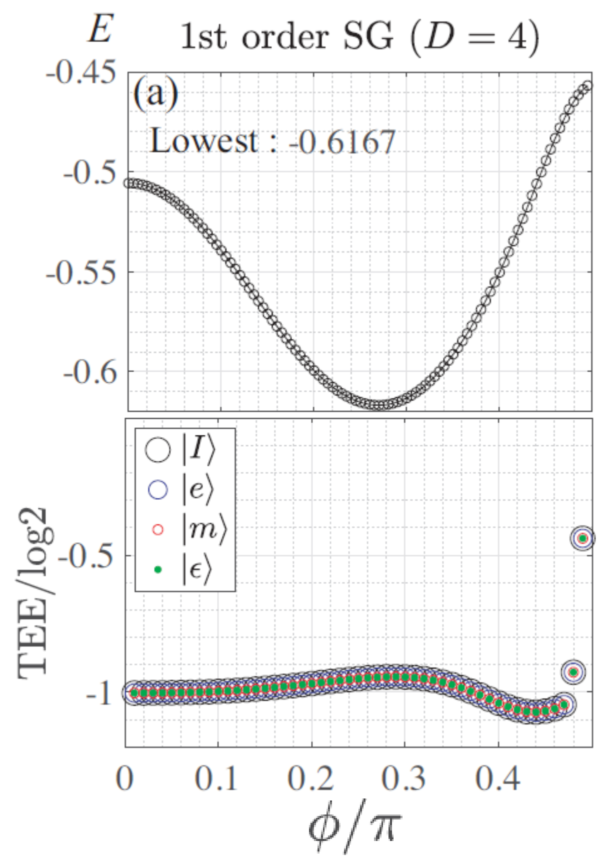
$\Rightarrow$ $\zeta(S=1) = 1/3 < \zeta_c = 1/\sqrt{3} = \zeta(S=1/2)$

S=1 loop-gas state is gapped.

Entanglement Entropy

MES





All minimally entangled states have the same topological entropy.

--> Abelian spin liquid

(Zhang et al: PRB85 235151 (2012))

Summary

New framework for describing Kitaev spin liquids

(1) S=1/2 Kitaev honeycomb model (KHM)

- both critical and gapped cases are expressed by TN
- KHM-LGS relation (analogous to AFH-AKLT relation)

(2) S=1/2 star-lattice Kitaev model

- both Abelian and non-Abelian gapped chiral SL are expressed as TN (CF: the no-go theorem by Dubail-Read (2015))
- Conformal tower of the Ising CFT

(3) S=1 Kitaev honeycomb model

- gapped Abelian SL

END