Towards computing the standard model of particle physics by tensor renormalization group

Yoshifumi Nakamura
RIKEN, R-CCS

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Dah-Hsian Seetoo Library/National Chengchi University
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Plan

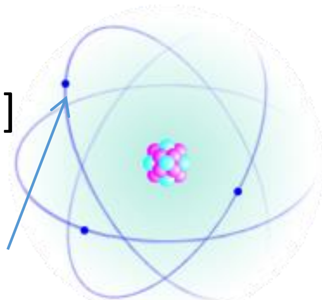
- **Standard model of particle physics**
- **Recent works for quantum field theories by TRG**
  - 2D complex ϕ^4 with chemical potential
  - 2D U(1) gauge theory with θ term
  - 2D free boson
  - 4D Ising model

Particle physics

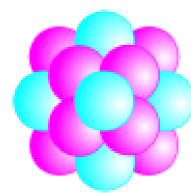
- **Study matter, motion in the universe**
  - Question since the beginning of human history
  - What is elementary (fundamental) particle

Atom
[10^{-10}m]

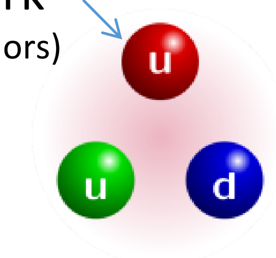
electron
(lepton)



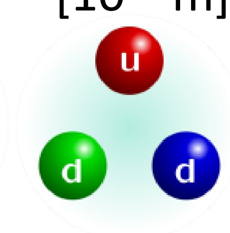
nucleus
[10^{-14}m]



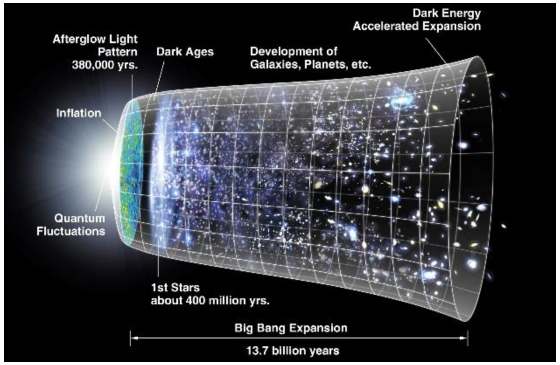
qaurk
(3 colors)



Proton/neutron
[10^{-15}m]



  - What is fundamental interaction?
  - How did the universe start, develop and become what it is today?
  - How will the universe be?



Quarks and leptons (fermions)

quarks : Components of hadrons, 6 flavors, 3 colors (red, blue, green)

up

Mass : 2.3 MeV/c^2
Charge : 2/3

charm

Mass : 1.275 GeV/c^2
Charge : 2/3

top

Mass : 173.07 GeV/c^2
Charge : 2/3

down

Mass : 4.8 MeV/c^2
Charge : -1/3

strange

Mass : 95 MeV/c^2
Charge : -1/3

bottom

Mass : 4.18 GeV/c^2
Charge : -1/3

leptons : charged leptons, neutrinos

electron neutrino

Mass : <2.2 eV/c^2
Charge : 0

muon neutrino

Mass : 0.17MeV/c^2
Charge : 0

tau neutrino

Mass : 15.5 MeV/c^2
Charge : 0

electron

Mass : 0.511MeV/c^2
Charge : -1

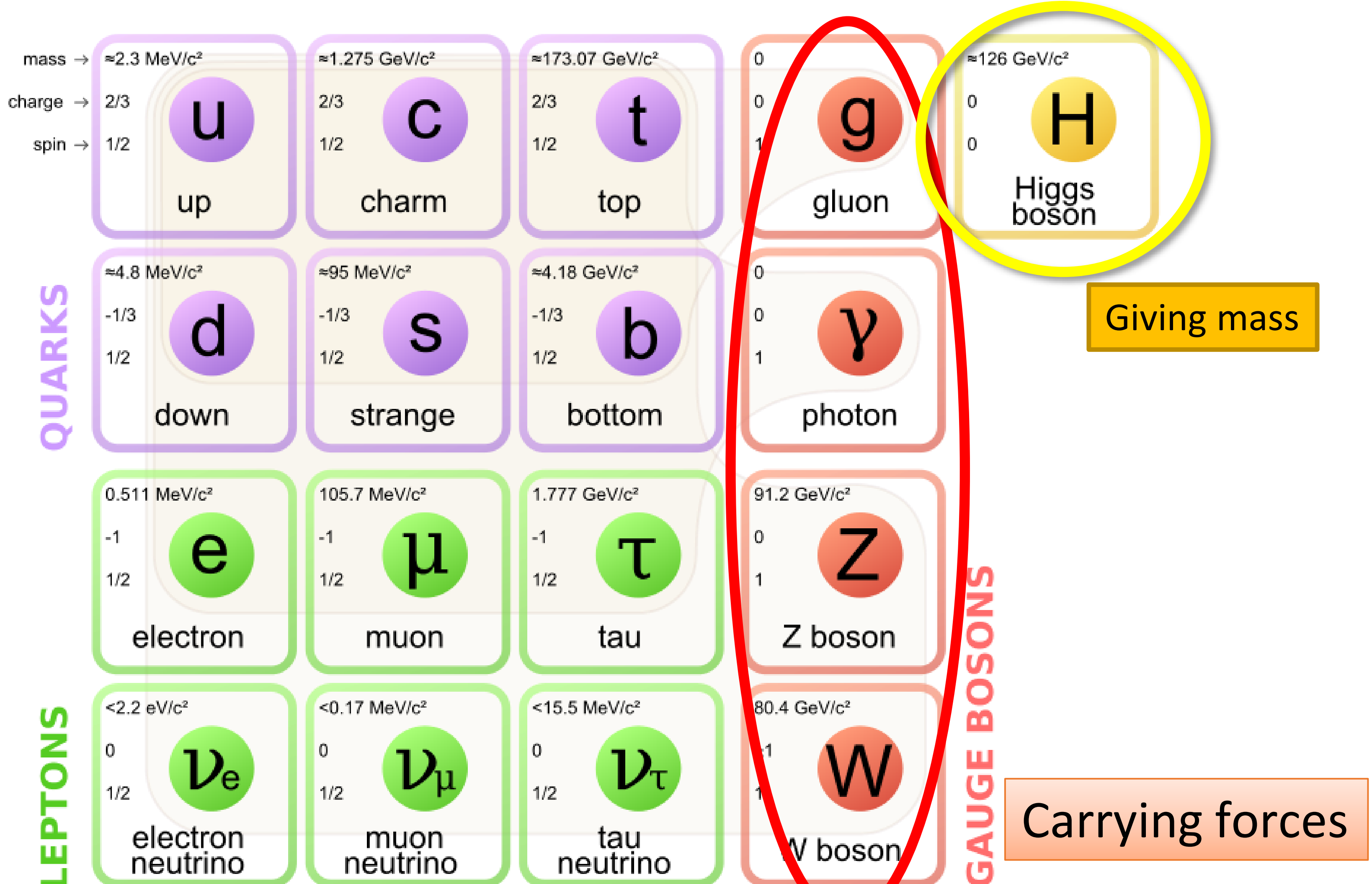
muon

Mass : 105.7MeV/c^2
Charge : -1

tau

Mass : 1.777GeV/c^2
Charge : -1

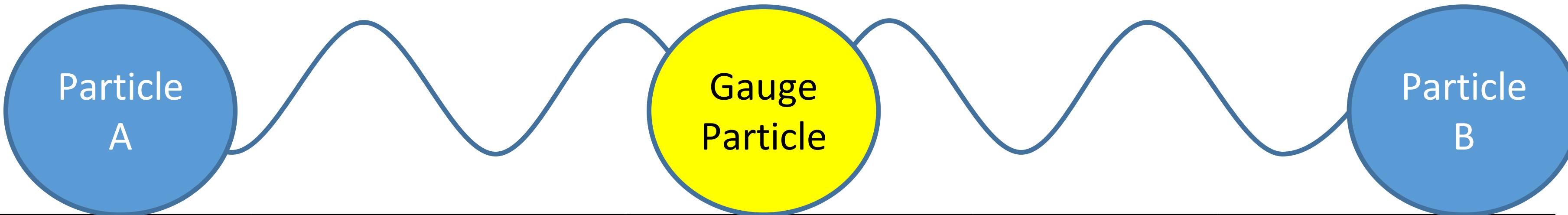
Elementary particles



wikipedia (Credit: MissMJ)

Four fundamental forces

Forces among particles (interaction) are carried by gauge particles



Force	Electromagnetic force	Weak force	Strong force	Gravity
Origin of force	electric charge	weak charge	color charge	mass
Range	∞	10^{-18}m	10^{-15}m	∞
Potential	$\frac{C}{r}$	$\frac{C_3}{r}e^{-m_W r}$	$\frac{C_1}{r}+C_2 r$	$\frac{C'}{r}$
Gauge boson	γ：photon	Z,W : weak boson	g: gluon	(graviton)
Classic theory	electromagnetism			General relativity
Quantum field theory	QED Electroweak theory (Glashow–Weinberg–Salam theory)		QCD	Not known (super string theory?)

$\underbrace{\text{Electromagnetic force} \quad \text{Weak force} \quad \text{Strong force}}_{\text{Standard model}}$

Standard model

Quantum field theory

- **Quantization of fields**
  - scalar boson, fermion, gauge boson
- **Gauge theory and symmetry**
  - describing interaction
  - U(1), SU(2), SU(3), ⋯ symmetries
- **Spontaneous symmetry breaking**
  - Higgs mechanism, chiral condensate
- **Lattice field theory**
  - the discretized theory of quantum field theory
  - Calculated numerically
- **Standard model (QED, EW theory, QCD), ϕ^4, Gross–Neveu, Schwinger model, + many more ...**

Sign problem on lattice field theory

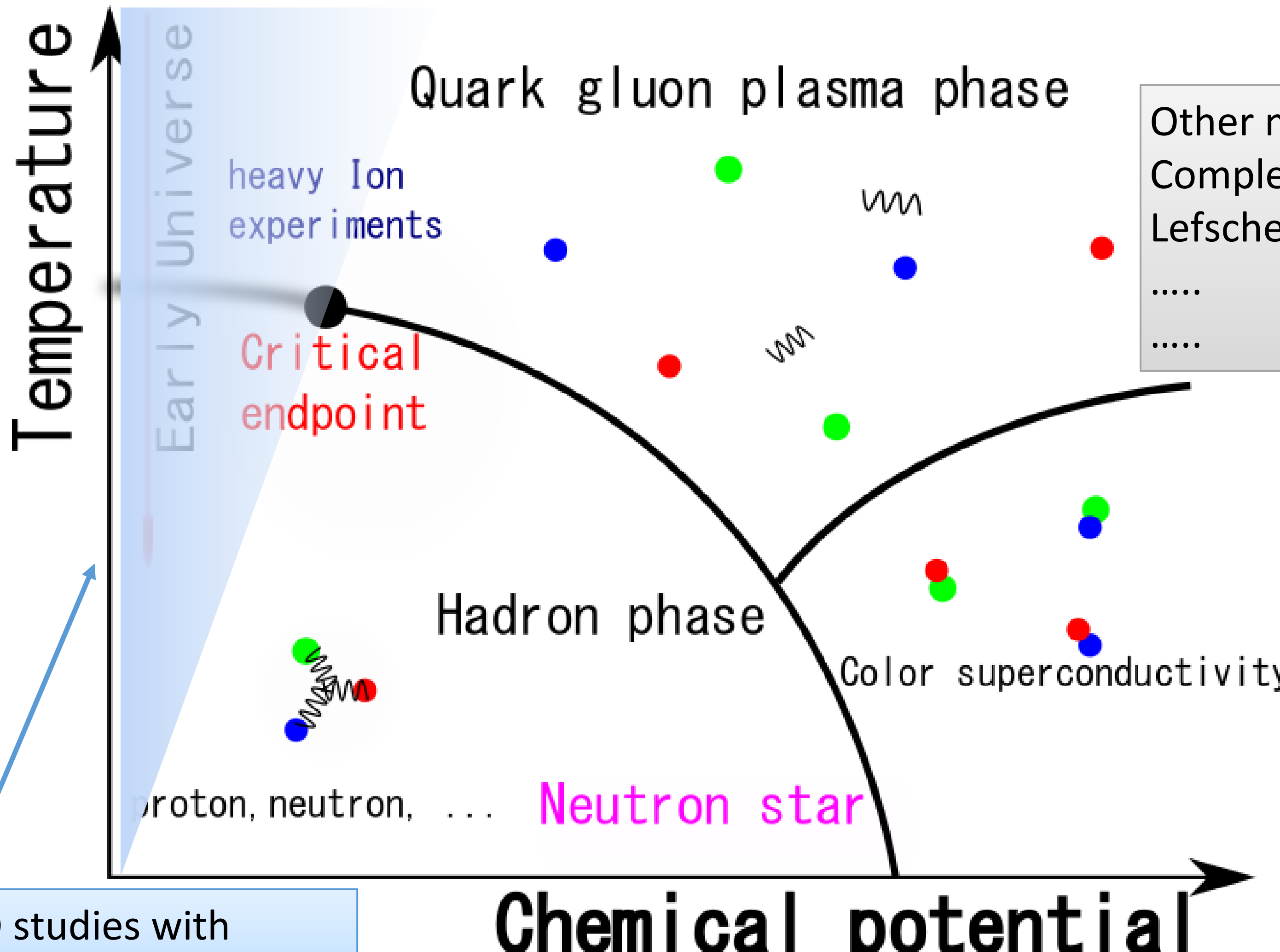
- **Monte Carlo simulations is powerful method to solve numerically quantum field theories on the lattice in no sign problem case**
- **Models suffering from the sign problem**
  - QCD with chemical potential
  - QCD with theta term
  - Chiral gauge theory
  - Models with chemical potential
  - Models with theta term
  - Supersymmetric models
  - …
  - …

Tensor network approach

Beyond standard models
Effective theories

QCD with chemical potential

QCD phase diagram



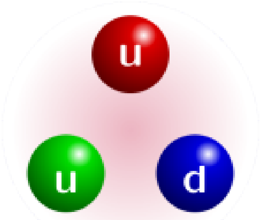
Other methods:
Complex Langevin
Lefschetz thimbles
.....
.....

Lattice QCD studies with Taylor expansion, reweighting, analytic continuation

Interesting physics at finite chemical potential

QCD with theta term

- **The two biggest unsolved problems of the strong interactions**



We can observe only color white particles

- Color Confinement
  - CMI Millennium Prize problem
- CP invariance
  - Strong CP problem

$$S = S_{QCD} + i\theta Q$$

Constraint from experiments and LQCD

$|\theta| < 10^{-10}$ or vanishing

New scalar particle (Axion?) solve the strong CP problem?
Peccei, Quinn, PRL38(1977)1440, PRD16 (1977)1791

Both problem relating?
Models suggest no confinement at $\theta \neq 0$
4D)Cardy,Rabinovici, NPB205(1982)1; Cardy, NPB205(1982)17
3D)Fradkin, Schaposnik PRL66(1991)276
2D)Coleman, Ann. of Phys. 101 (1976) 239

Field treatment on tensor network

- **We need to treat scalar, gauge, and fermion fields**
- **Obtaining finite dimensional tensor network from action containing continuous variable (Lagrangian TN)**
  - Scalar field
    - Orthogonal function expansion : Shimizu, Mod.Phys.Lett.A27(2012)1250035
    - Gauss-Hermite quadrature : Kadoh et al., JHEP03(2018)141
    - Gaussian SVD/TRG : Campos et al., PRB100(2019)195106
  - Gauge field
    - Character expansion : Liu et al., PRD88(2013)056005
    - Gauss-Legendre quadrature: Kuramashi, Yoshimura, arXiv:1911.06480
  - Fermion field
    - Grassmann TRG : Shimizu, Kuramashi, PRD90(2014)014508, Takeda, Yoshimura, PTEP2015(2015)043B01

2D Complex ϕ^4 with chemical potential

Kadoh, Kuramashi, YN, Takeda, Sakai, Yoshimura
in preparation

$$S = \int d^2x \left\{ \partial^\nu \phi^* \partial_\nu \phi + (m^2 - \mu^2)|\phi|^2 + \mu(\phi_2^* \phi + \partial_2 \phi^* \phi) + \lambda |\phi|^4 \right\}$$

- **m, μ, λ : mass, chemical potential, quartic coupling constant**
- **ϕ : complex scalar field**
- **Suffering from sign problem**

Lattice partition function

$$Z = \int \mathcal{D}\phi e^{-S}$$

$$S = \sum_n \left[(4 + m^2)|\phi_n|^2 + \lambda |\phi_n|^4 - \sum_{\nu=1}^{2} (e^{\mu \delta_{\nu,2}} \phi_n^* \phi_{n+\hat{\nu}} + e^{-\mu \delta_{\nu,2}} \phi_{n+\hat{\nu}}^* \phi_n) \right]$$

2D Complex ϕ^4 tensor network

Kadoh, Kuramashi, YN, Takeda, Sakai, Yoshimura
in preparation

$$e^{-S} = \prod_n \prod_\nu f_\nu(\phi_n, \phi_{n+\hat{\nu}})$$

$$f_\nu(\phi_n, \phi_{n+\hat{\nu}}) = \exp\left\{ -\frac{1}{4}(4-m^2)(|\phi_n|^2 + |\phi_{n+\hat{\nu}}|^2) + \frac{\lambda}{4}(|\phi_n|^4 + |\phi_{n+\hat{\nu}}|^4) + e^{\mu\delta_{\nu,2}}\phi_n^*\phi_{n+\hat{\nu}} + e^{-\mu\delta_{\nu,2}}\phi_{n+\hat{\nu}}^*\phi_n \right\}$$

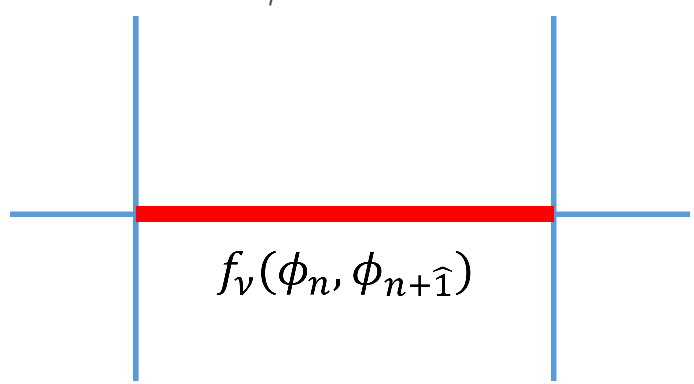
Gauss-Hermite quadrature of $\phi = \frac{1}{\sqrt{2}}(\phi_R + i\phi_I)$

$$\int_{-\infty}^{\infty} d\phi_R \int_{-\infty}^{\infty} d\phi_I f_\nu(\phi_n, \phi_{n+\hat{\nu}}) \approx \sum_{\alpha=1}^{K} \sum_{\beta=1}^{K} \omega_\alpha \omega_\beta e^{x_\alpha^2 + x_\beta^2} f_\nu(x_n, x_{n+\hat{\nu}})$$

$K^2 \times K^2$ matix components

$$x = \frac{1}{\sqrt{2}}(x_\alpha + i x_\beta)$$

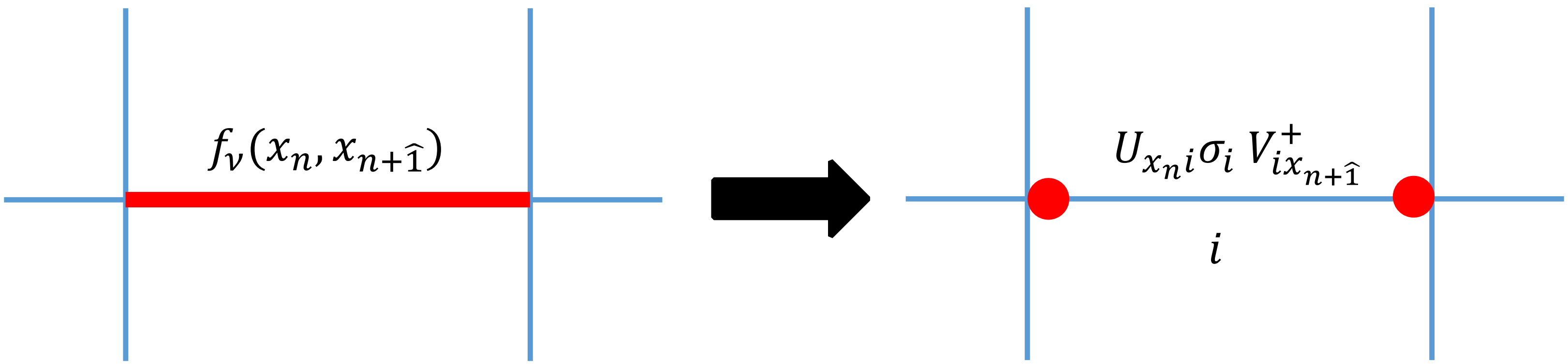
K : degree
ω : weights
x : nodes



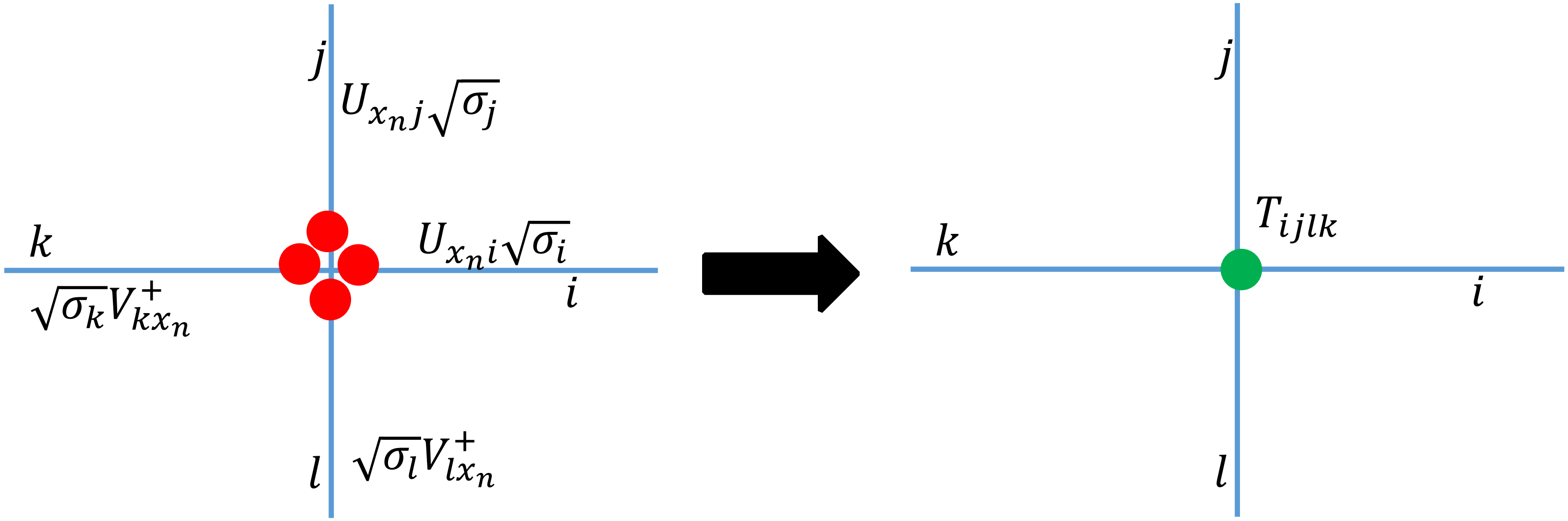
2D Complex ϕ^4 tensor network

Kadoh, Kuramashi, YN, Takeda, Sakai, Yoshimura
in preparation

SVD : $f_\nu(x_n, x_{n+\hat{1}}) = \Sigma_i\, U_{x_n i}\sigma_i\, V^+_{i x_{n+\hat{1}}}$



SVD for other directions and sum over x including weight/node factors of GH



Average phase factor

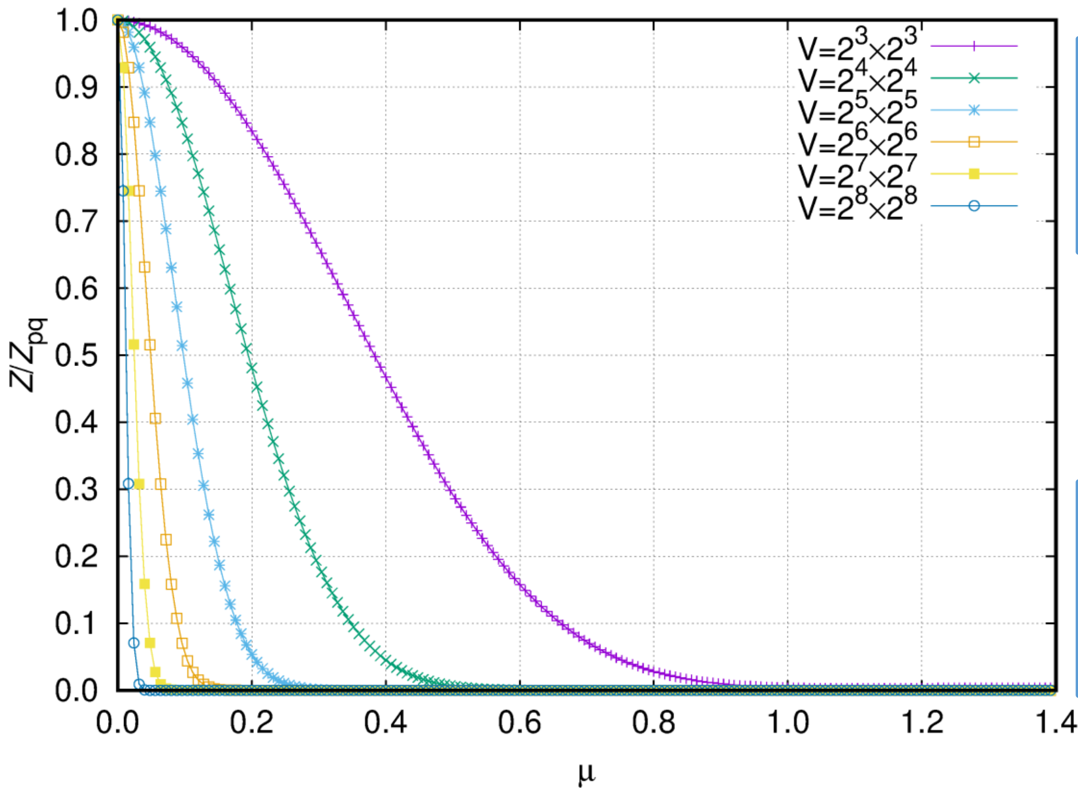
$m^2 = 0.01, \lambda = 1, K = 64,$
Bond dimension $D = 64$

Kadoh, Kuramashi, YN, Takeda, Sakai, Yoshimura
in preparation

Phase reweighting

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{O} e^{-S}}{Z} = \frac{\int \mathcal{O} e^{i\theta} |e^{-S}|}{\int e^{i\theta} |e^{-S}|} = \frac{\int \mathcal{O} e^{i\theta} |e^{-S}|}{\int |e^{-S}|} \frac{\int |e^{-S}|}{\int e^{i\theta} |e^{-S}|}$$

$$= \frac{\int \mathcal{O} e^{i\theta} |e^{-S}|}{Z_{pq}} \frac{Z_{pq}}{\int e^{i\theta} |e^{-S}|} = \frac{\langle \mathcal{O} e^{i\theta} \rangle_{pq}}{\langle e^{i\theta} \rangle_{pq}}$$



If average phase factor is ~0, one can't obtain signal in the Monte Carlo simulations

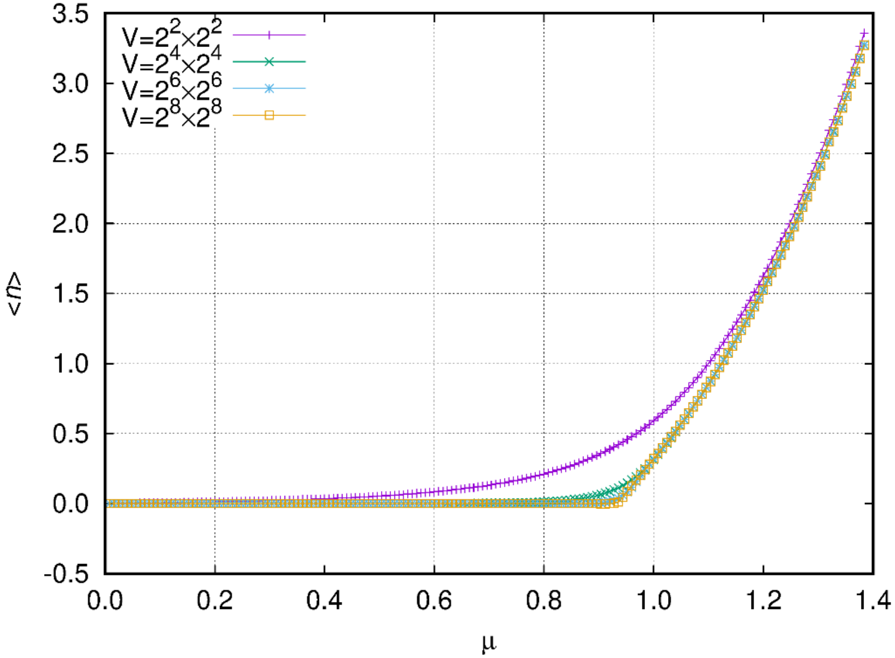
Average phase factor is ~0 at large volume and chemical potential in this system

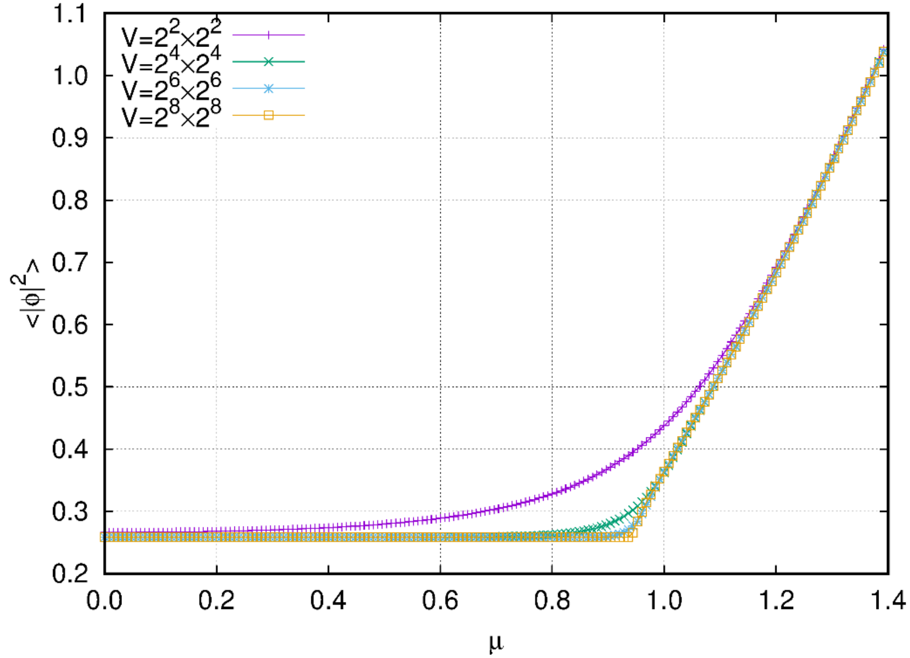
Silver blaze phenomenon

$m^2 = 0.01, \lambda = 1, K = 64,$
Bond dimension $D = 64$

Kadoh, Kuramashi, YN, Takeda, Sakai, Yoshimura
in preparation

Observables do not depend on the chemical potential below the critical point





Particle number density

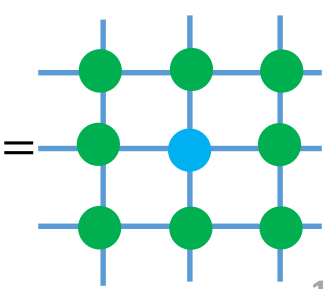
$$\langle n \rangle = \frac{1}{N_s N_T} \frac{\partial \ln Z}{\partial \mu}$$

Numerical derivative

$$\langle |\phi|^2 \rangle = \frac{Z'}{Z}$$

Impure tensor method

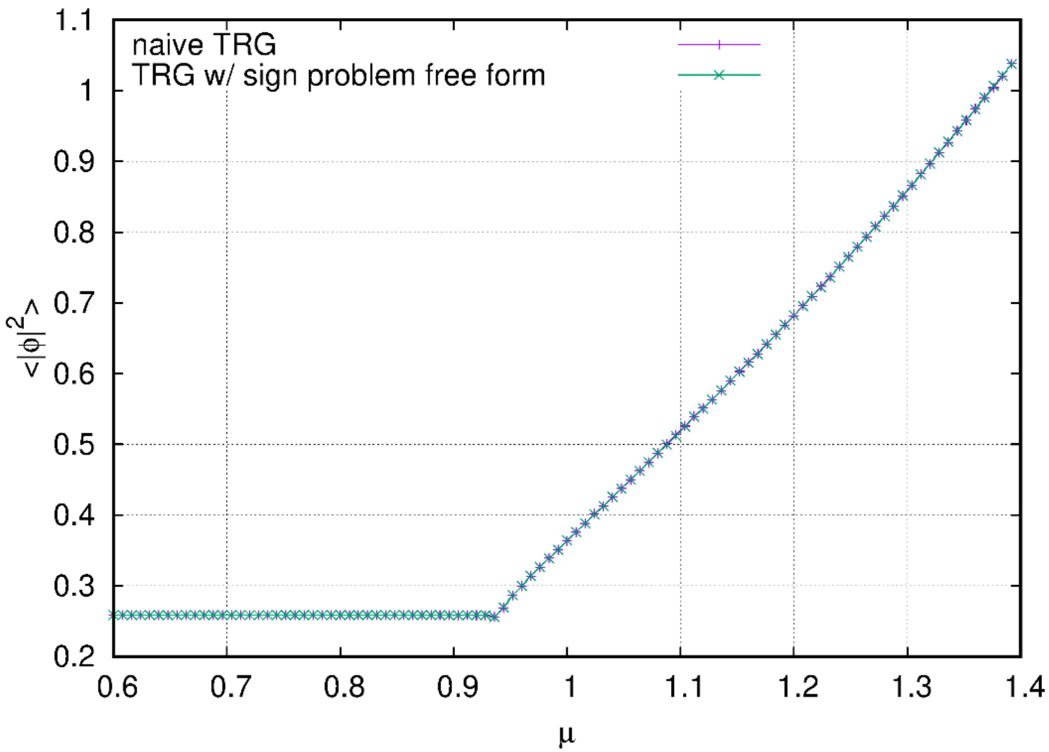
$Z' =$



Comparing with sign problem free form

$m^2 = 0.01, \lambda = 1, K = 64,$
Bond dimension $D = 64$
$V = 2^{10} \times 2^{10}$

Kadoh, Kuramashi, YN, Takeda, Sakai, Yoshimura
in preparation



Sign problem free form (real action)
Endres, PoS(LAT2006)133,
PRD75(2007)065012
Integrating out angular modes using character expansion after expressing polar coordinate

Good agreement : TRG works in system with sign problem in MC

2D U(1) gauge theory with θ term

Kuramashi, Yoshimura, arXiv:1911.06480

$$S = -\beta \sum_x \cos p_x - i\theta Q$$

$$p_x = \varphi_{x,1} + \varphi_{x+\hat{1},2} - \varphi_{x+\hat{2},1} - \varphi_{x,2}$$

$$Q = \frac{1}{2\pi} \sum_x q_x, \quad q_x = p_x \bmod 2\pi$$

$$\varphi_{x,\mu} \in [-\pi, \pi]$$

- **β, θ : coupling constant, vacuum angle**
- **Suffering from sign problem**
- **Phase transition at $\theta = \pi$ in the strong coupling limit. [Seiberg,PRL53(1984)637]**

Partition function

$$Z = \left(\prod_{x,\mu} \int_{-\pi}^{\pi} \frac{d\varphi_{x,\mu}}{2\pi} \right) \prod_x \mathcal{T}(\varphi_{x,1}, \varphi_{x+\hat{1},2}, \varphi_{x+\hat{2},1}, \varphi_{x,2})$$

$$\mathcal{T}(\varphi_{x,1}, \varphi_{x+\hat{1},2}, \varphi_{x+\hat{2},1}, \varphi_{x,2}) = \exp\left(\beta \cos p_x + i \frac{\theta}{2\pi} q_x \right)$$

Tensor form partition function

$$Z \approx \sum_{\{\alpha\}} \prod_x T_{\alpha_{x,1} \alpha_{x+\hat{1},2} \alpha_{x+\hat{2}} \alpha_{x,2}}$$

$$T_{ijkl} = \frac{\sqrt{w_i w_j w_k w_l}}{(2\pi)^2} \mathcal{T}\left(\varphi^{(i)}, \varphi^{(j)}, \varphi^{(k)}, \varphi^{(l)} \right)$$

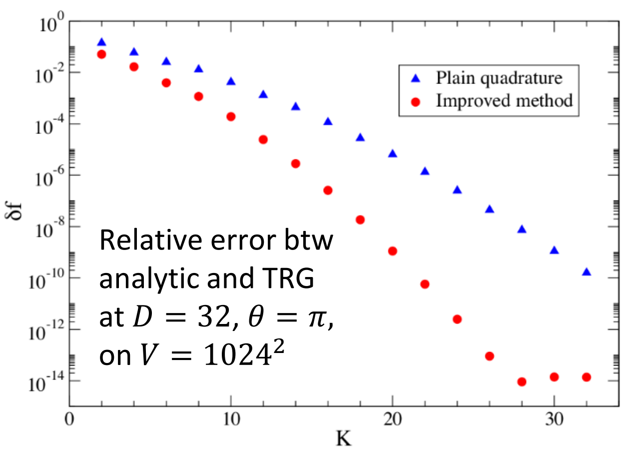
Gauss-Legendre quadrature

$$\int d\varphi f(\varphi) \approx \sum_{\alpha=1}^{K} w_\alpha f\left(\varphi^{(\alpha)}\right)$$

$$Z_{\mathrm{analytic}} = \sum_{Q=-\infty}^{\infty} \left(z_{\mathrm{P}}(\theta + 2\pi Q, \beta) \right)^V$$

$$z_{\mathrm{P}}(\theta, \beta) = \int_{-\pi}^{\pi} \frac{d\varphi_{\mathrm{P}}}{2\pi} \exp\left(\beta \cos \varphi_{\mathrm{P}} + i \frac{\theta}{2\pi} \varphi_{\mathrm{P}} \right)$$

Wiese, NPB318(1989)153

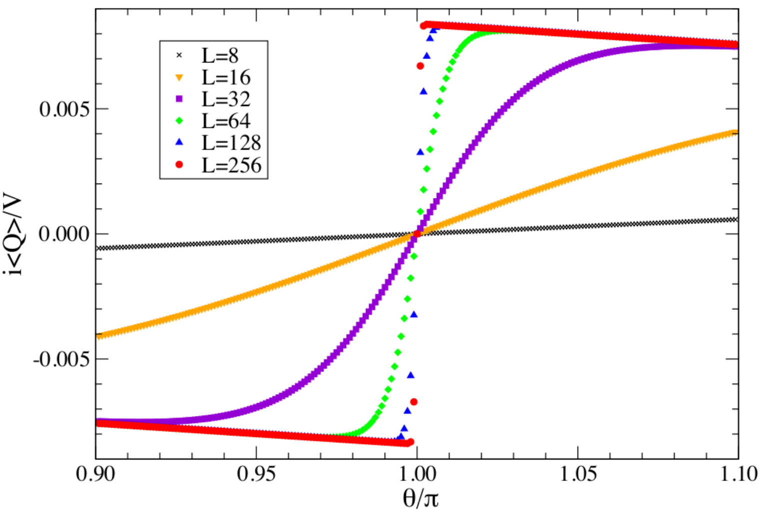


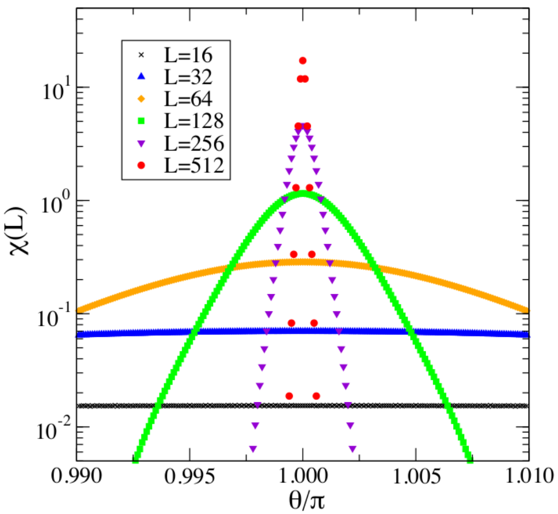
Improvement using character expansion for initial tensor leads to machine precision

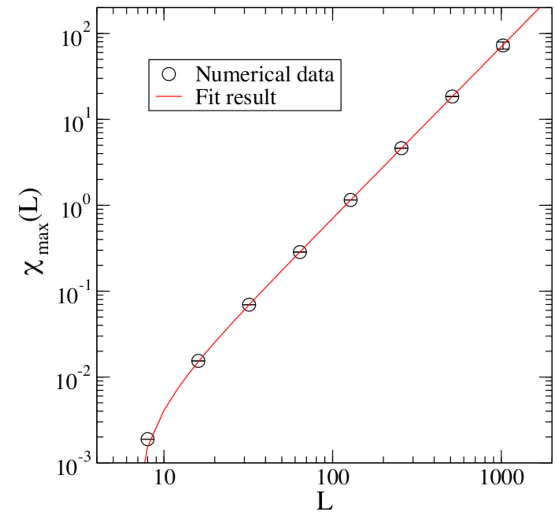
2D U(1) gauge theory with θ term

$\boxed{\beta = 10, D = 32, K = 32}$

Kuramashi, Yoshimura, arXiv:1911.06480







$$\langle Q\rangle = -i\frac{\partial \ln Z}{\partial\theta}$$

$$\chi(L) = -\frac{1}{V}\frac{\partial^2 \ln Z}{\partial\theta^2}$$

$$\chi_{\max}(L) \propto L^{\gamma/\nu}$$

$$\frac{\gamma}{\nu} = 1.998(2)$$

1st order phase transition

TRG works in system with sign problem in MC

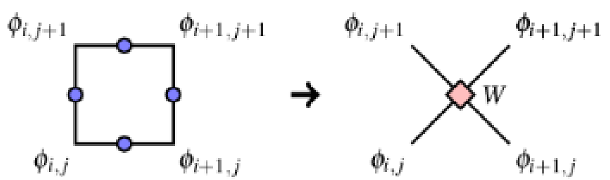
2D free boson

Lattice partition function

Campos, Sierra, Lopez, PRB100(2019)195106

$$Z = \int \prod_{ij} d\phi_{ij}\; e^{-\frac{1}{2}\sum_{ij}\left[(\phi_{ij}-\phi_{i+1j})^2+(\phi_{ij}-\phi_{ij+1})^2+m^2\phi_{ij}^2\right]}$$

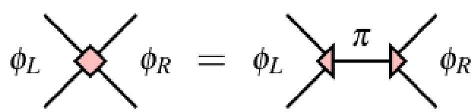
Vertex form



$$W(\phi_i) = e^{-\frac{1}{2}\sum_{i=1}^{4}\left[(\phi_i-\phi_{i+1})^2+\frac{m^2}{2}\phi_i^2\right]}$$

Origiral TRG
(Orthogonal func. exp.)
$W = UDV^+$
Shimizu (2012)

Gaussian SVD(TRG) with using new fields



$$W = \rho\, e^{-\frac{1}{2}\phi_L^T A_L \phi_L - \frac{1}{2}\phi_R^T A_R \phi_R + \phi_L^T B \phi_R}$$

ρ : normalize constant
A_L, A_R, B : real matrix

Using SVD of B
$B = UDV^+$

$$W(\phi_L,\phi_R) = G_L(\phi_L)\,\widehat{W}(\phi_L,\phi_R)\,G_R(\phi_R)$$

$$\widehat{W} = \int d\pi\; e^{i\phi_L^T U\pi}\, S(\pi)\, e^{-i\pi^T V^T\phi_R}$$

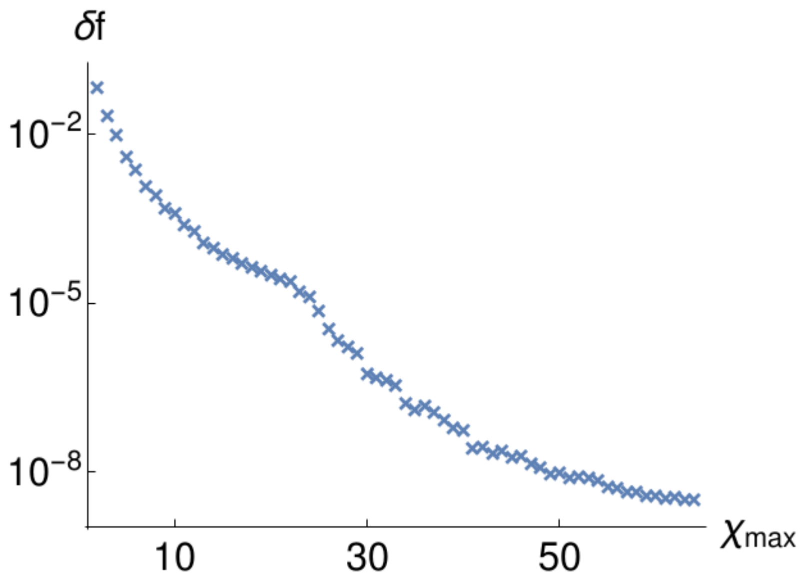
$$S = \frac{1}{\sqrt{(2\pi)^{\tilde{\chi}}\det D}}\, e^{-\frac{1}{2}\pi^T D^{-1}\pi}$$

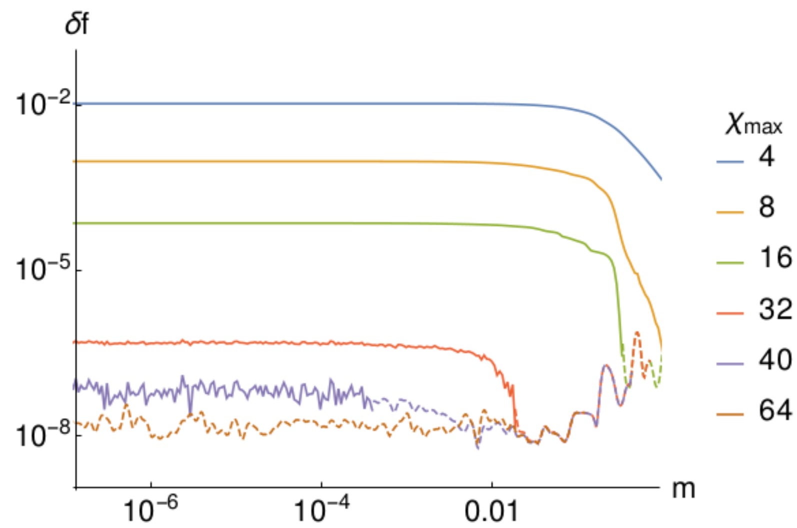
$$G_L = e^{-\frac{1}{2}\phi_L^T(A_L-UDU^T)\phi_L}\,,\quad G_R = e^{-\frac{1}{2}\phi_R^T(A_R-VDV^T)\phi_R}$$

2D free boson

Campos, Sierra, Lopez, PRB100(2019)195106

$\boxed{L = 2^{30}, m = 1.2 \times 10^{-6}}$



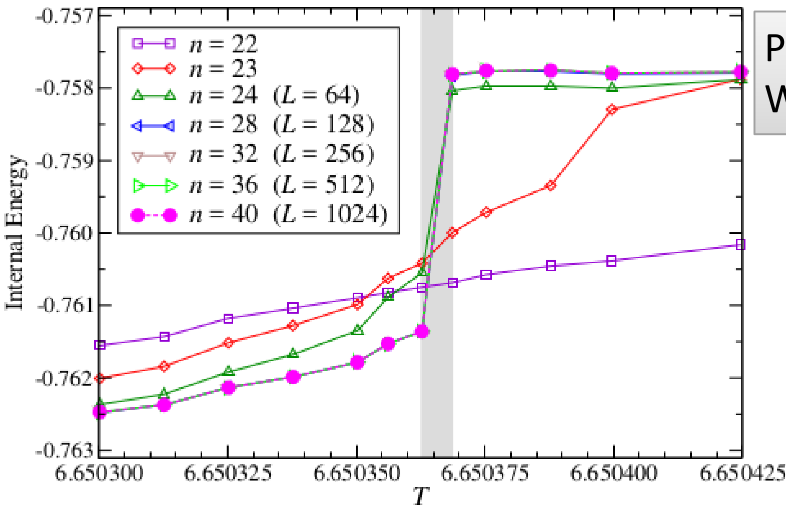


Relative error for free energy is small at small mass
Central charge agrees to theoretical value, 1, at massless limit
How about Interacting case?

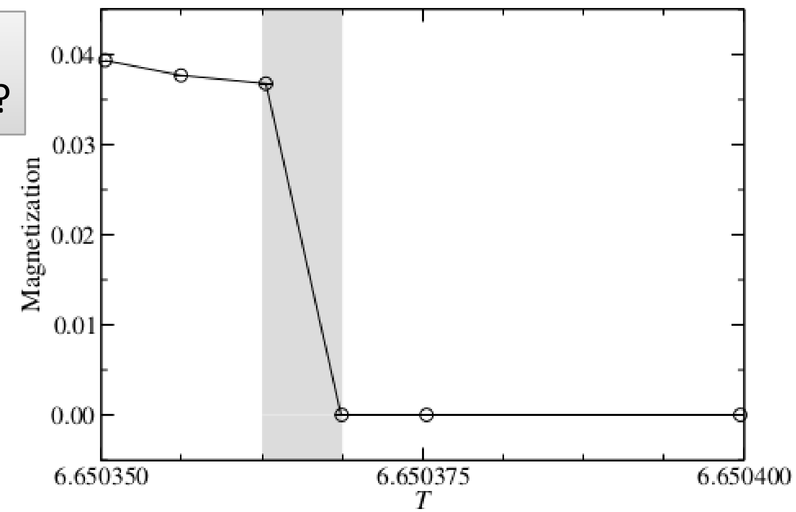
4D Ising model (HOTRG with $D = 13$)

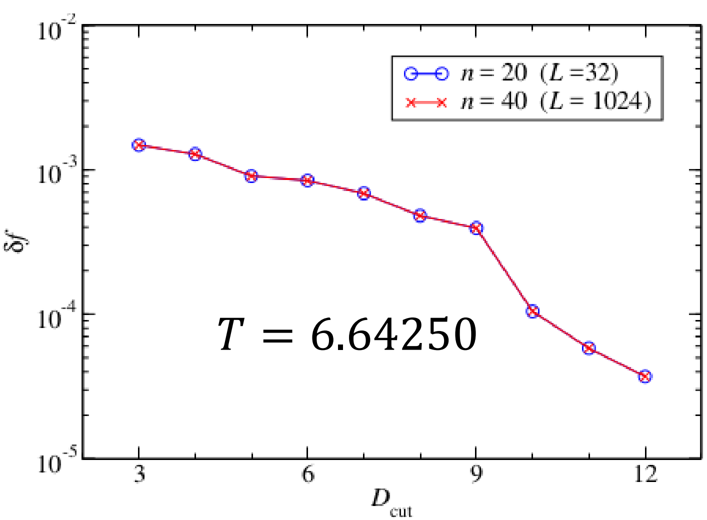
Akiyama, Kuramashi, Yamashita, Yoshimura, PRD100(2019)054510

Computational cost $O(D^{13})$/process : HOTRG(cost:$O(D^{15})$ on D^2 processes



Phase transition
Weak first order?





$$\delta f = \left| \frac{\ln Z_N(D) - \ln Z_N(D=13)}{\ln Z_N(D=13)} \right|$$

Cost reduction is very important

ATRG : Adachi, Okubo, Todo, arXiv:1906.02007

$O\left(D^{2d+1}\right)$

ATRG improvement : Oba, arXiv:1908.07295

Swapping bond part : $\mathrm{O}\left(D^{\max(d+3,7)}\right)$

NEW : talk by Kadoh at Dec. 6, TNSAA7 2019-2020

Summary

- **I introduced the standard model of particle physics and two sign problem systems on standard model**
  - QCD with chemical potential
  - QCD with θ term
- **Recent works for quantum field theories by TRG**
  - 2D complex ϕ^4 with chemical potential
  - 2D U(1) gauge theory with θ term
  - 2D free boson
  - 4D Ising model
- **Future studies for SM**
  - non-Abelian gauge theories
  - 4D systems with better algorithm on massively parallel machines (e.g. 150k+ nodes (600k+ proc.) supercomputer Fugaku)