Anisotropic tensor renormalization group and BTRG

The University of Tokyo, Tsuyoshi Okubo

Ref. D. Adachi, T. Okubo, and S. Todo, arXiv:1906.02007

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- Bond-weighted TRG: **B**TRG....
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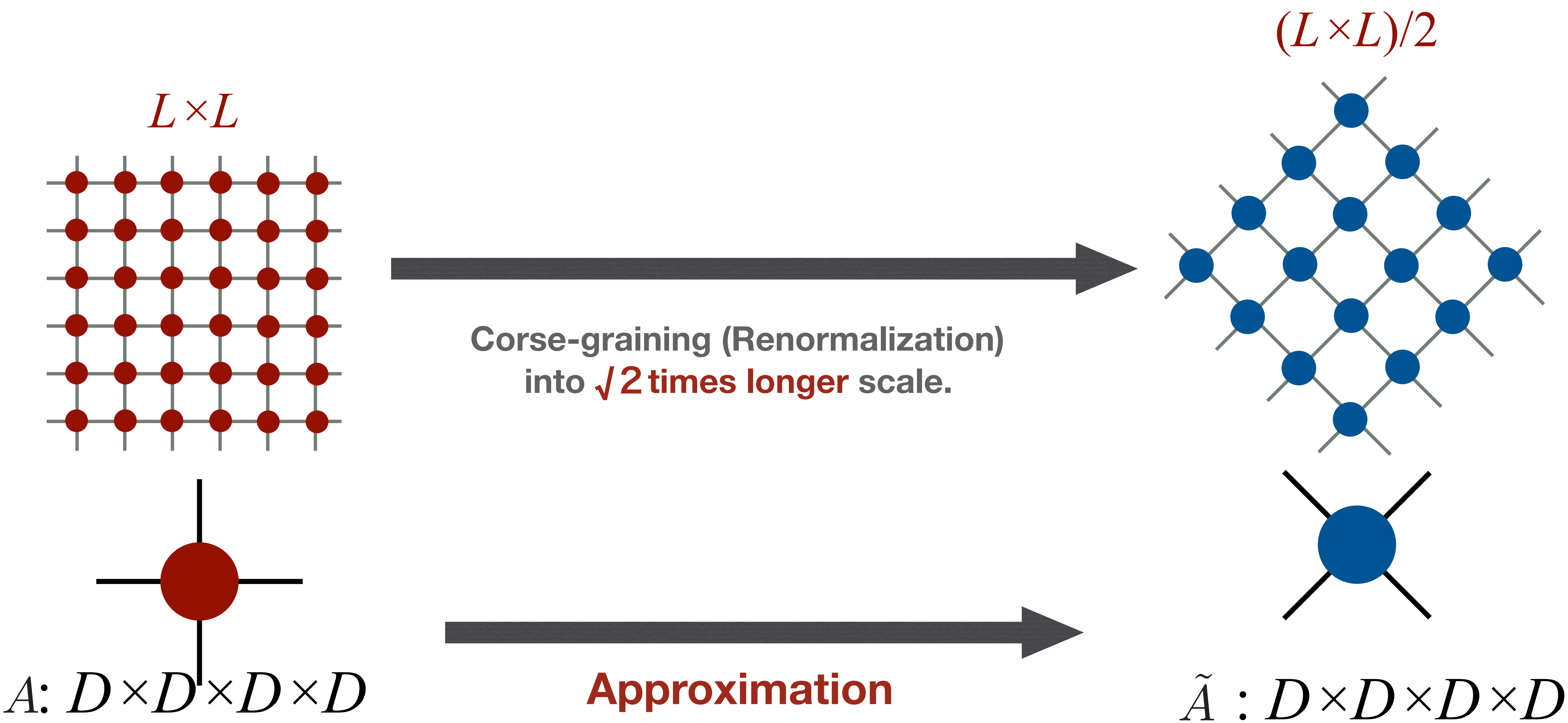
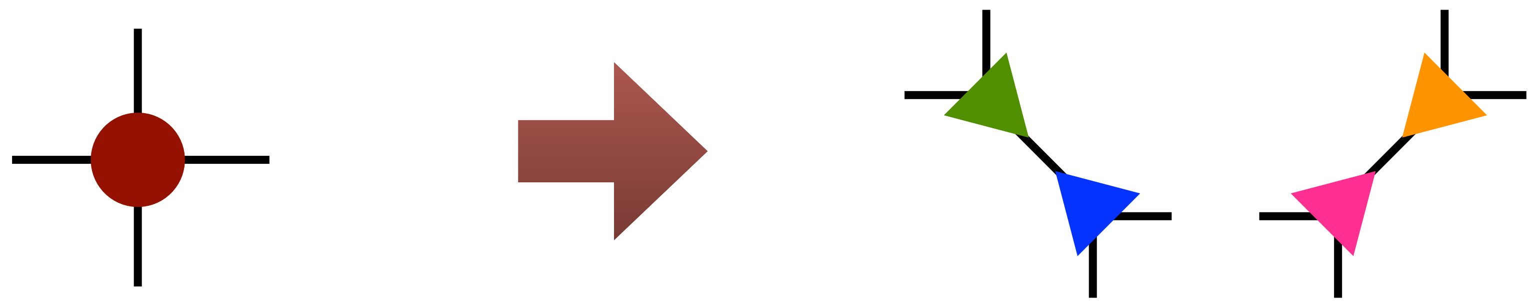
Tensor network renormalization group

Purpose: approximate contraction of tensor network by using "coarse-graining" of the network



Tensor Renormalization Group (TRG) algorithm

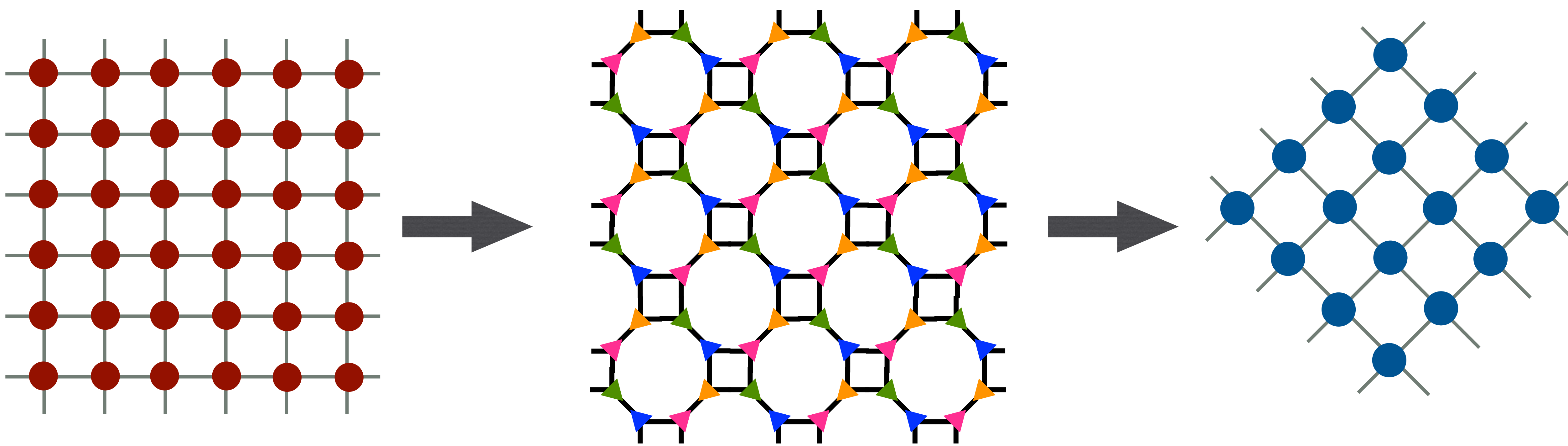
TRG M. Levin and C. P. Nave, Phys. Rev. Lett. **99**, 120601 (2007)



Low rank approximation by SVD

Computation cost: $O(D^5)$

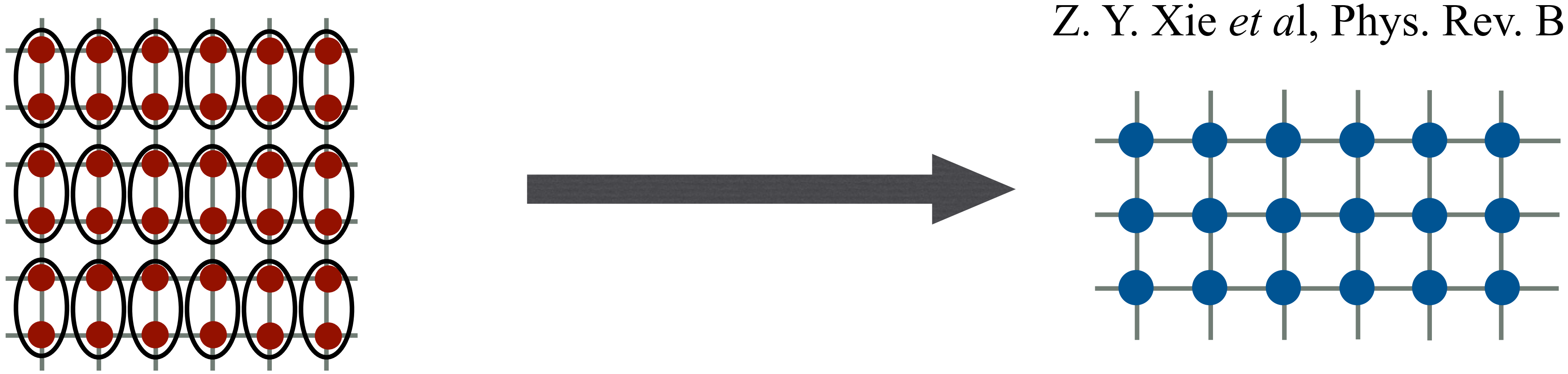
Memory: $O(D^3)$



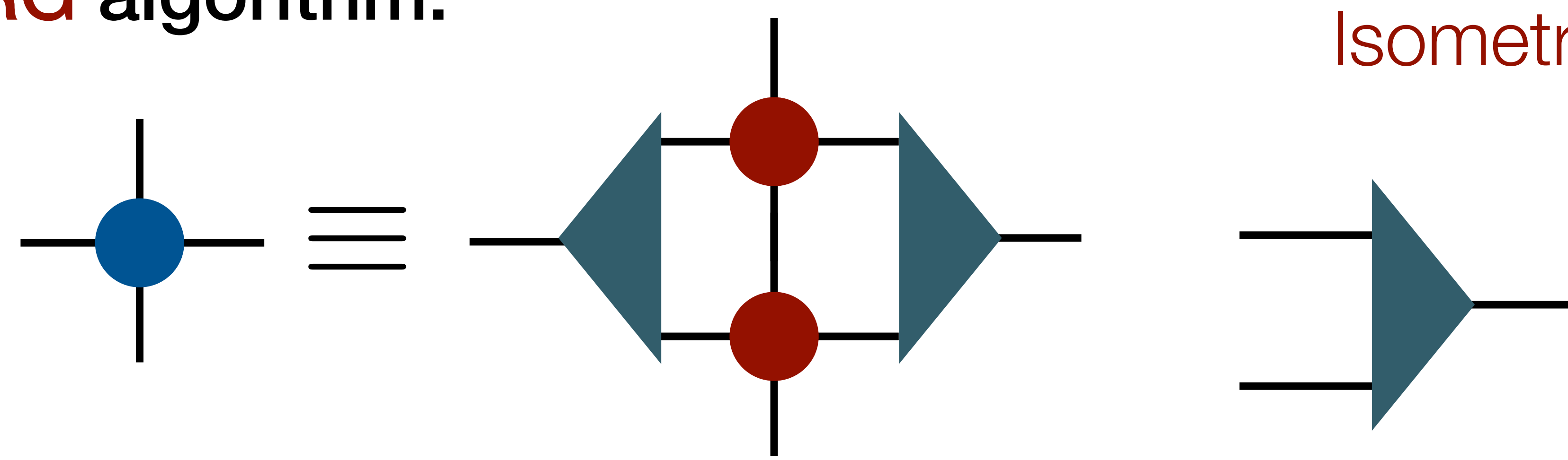
Higher Oder Tensor Renormalization Group (HOTRG)

Anisotropic coarse-graining by using HOSVD instead of SVD

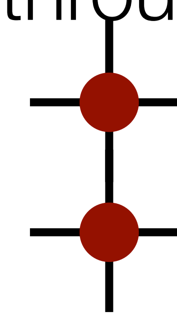
Z. Y. Xie *et al*, Phys. Rev. B **86**, 045139 (2012)



HOTRG algorithm:



Isometry is defined through HOSVD of



Better accuracy than TRG, although,

Computation cost: $O(D^7) > O(D^5)$ (TRG)

Application to high dimensions

Interests in

- 3d classical systems
- 2d and 3d quantum systems
- Much higher dimensions...

We want to perform tensor network RG for high dimensions!

However,

TRG: Not easy to generalize to high dimensions.

HOTRG: Easy to generalize to high dimensions, but its cost is $O(D^{4d-1})$

$d=3 : O(D^{11})$

$d=4 : O(D^{15})$

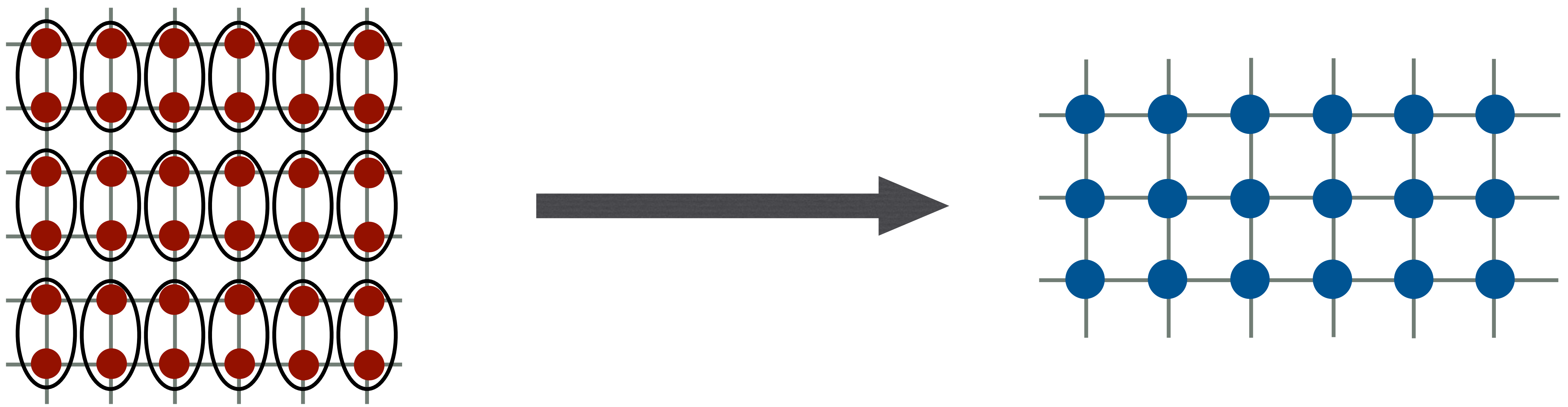
Is it possible to construct lower cost algorithm ?

Anisotropic TRG = ATRG

D. Adachi, T. Okubo, and S. Todo, arXiv:1906.02007

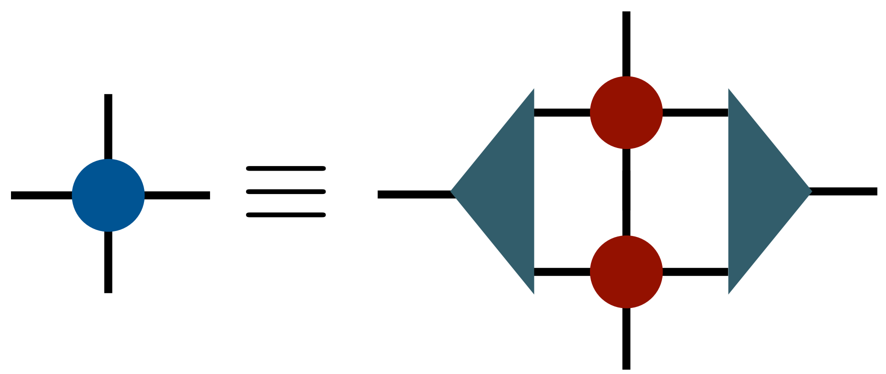
Central idea of Anisotropic TRG

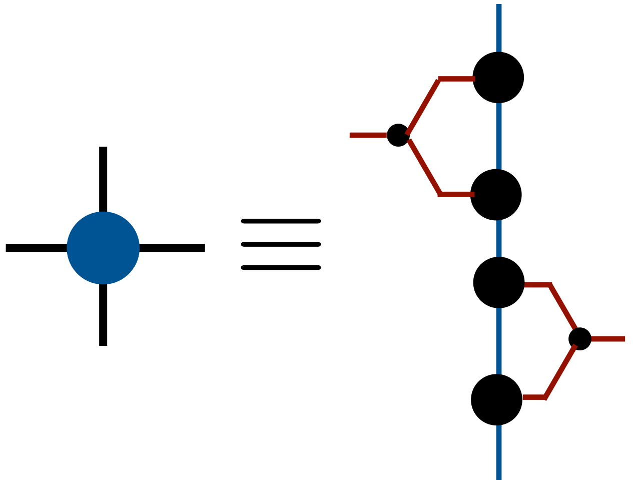
In ATRG, we coarse-grain tensors anisotropically as HOTRG:



In order to reduce the computation cost, we decompose the local tensor into small pieces before performing coarse-graining.

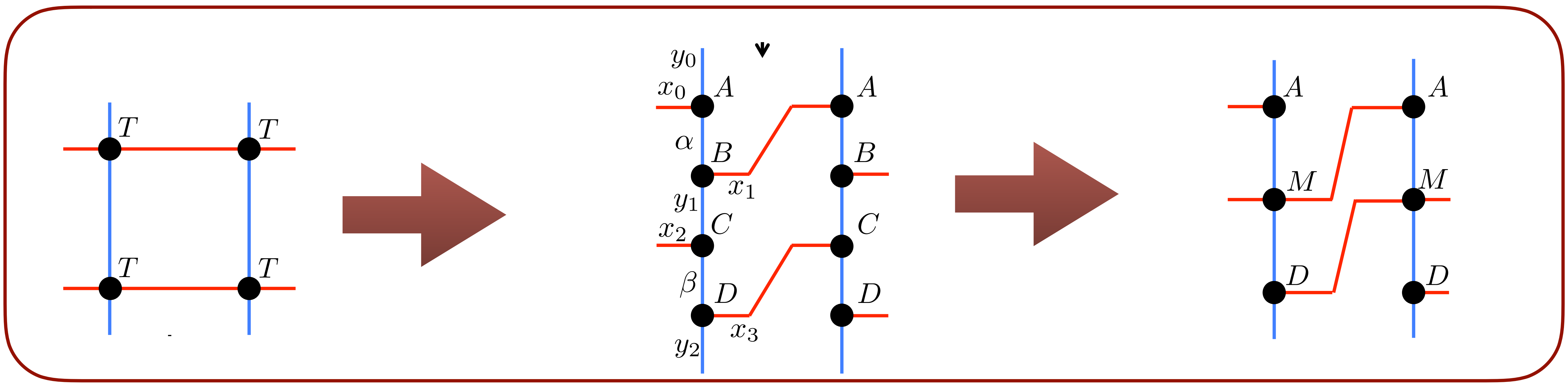
HOTRG $O(D^7)$ **ATRG** $O(D^5)$

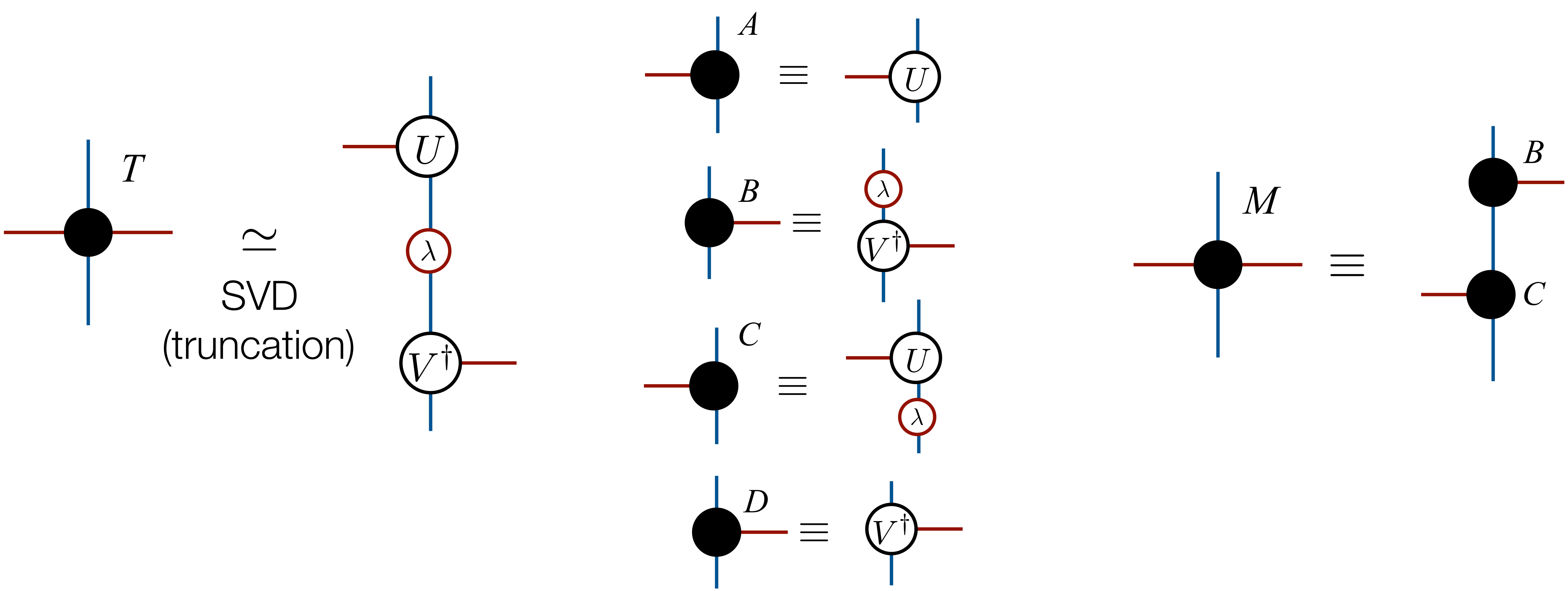




Recipe of ATRG

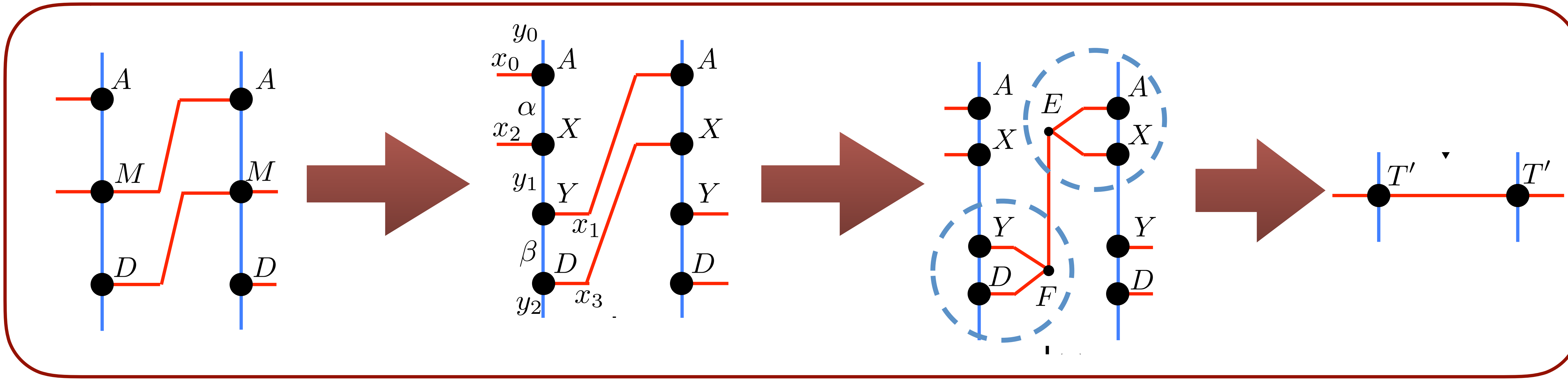
D. Adachi, T. Okubo, and S. Todo, arXiv:1906.02007

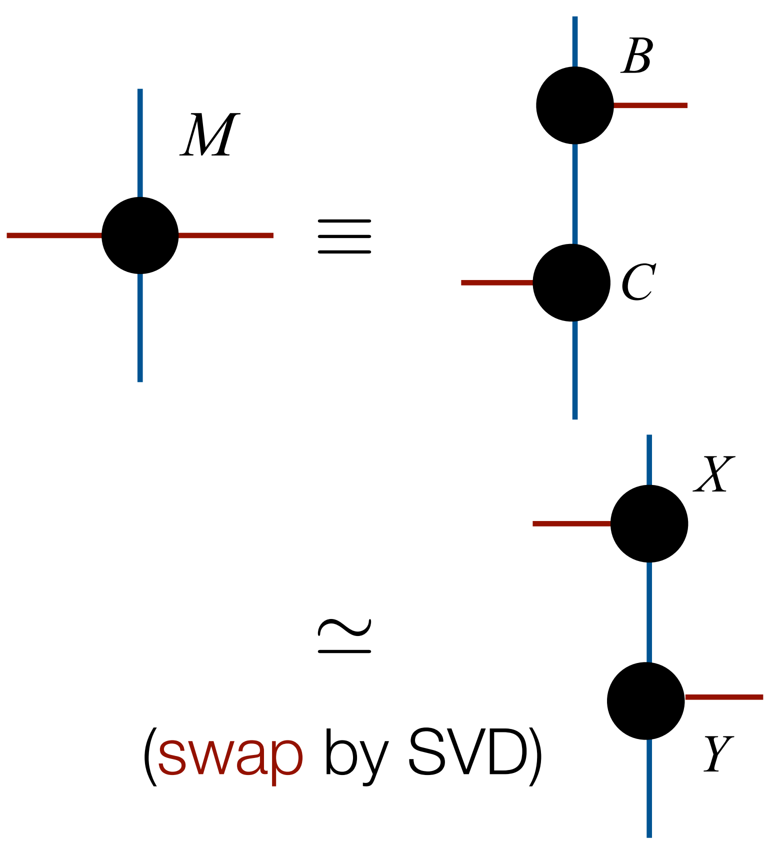


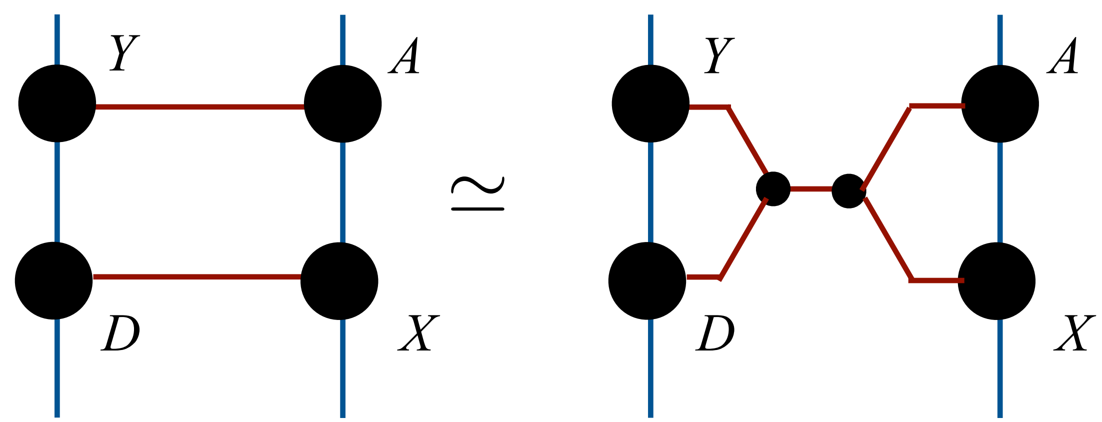


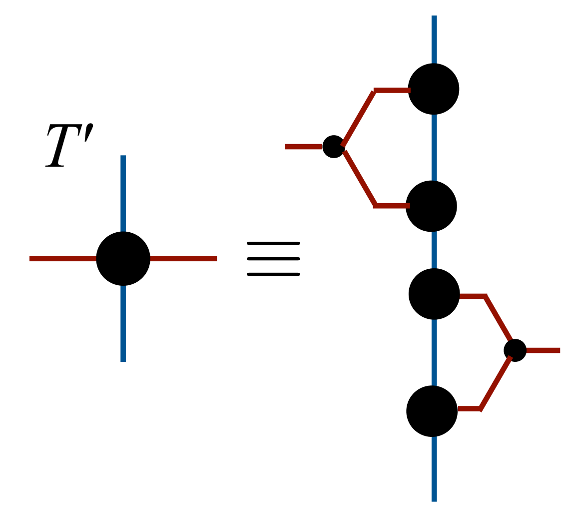
Recipe of ATRG

D. Adachi, T. Okubo, and S. Todo, arXiv:1906.02007









Inserte projector

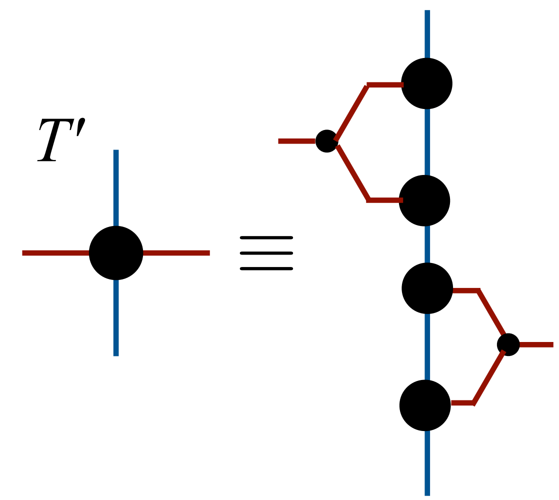
Different from HOTRG,
this step can be done with $O(D^5)$

Summary of 2D ATRG

D. Adachi, T. Okubo, and S. Todo, arXiv:1906.02007

Memory storage: $O(D^3)$

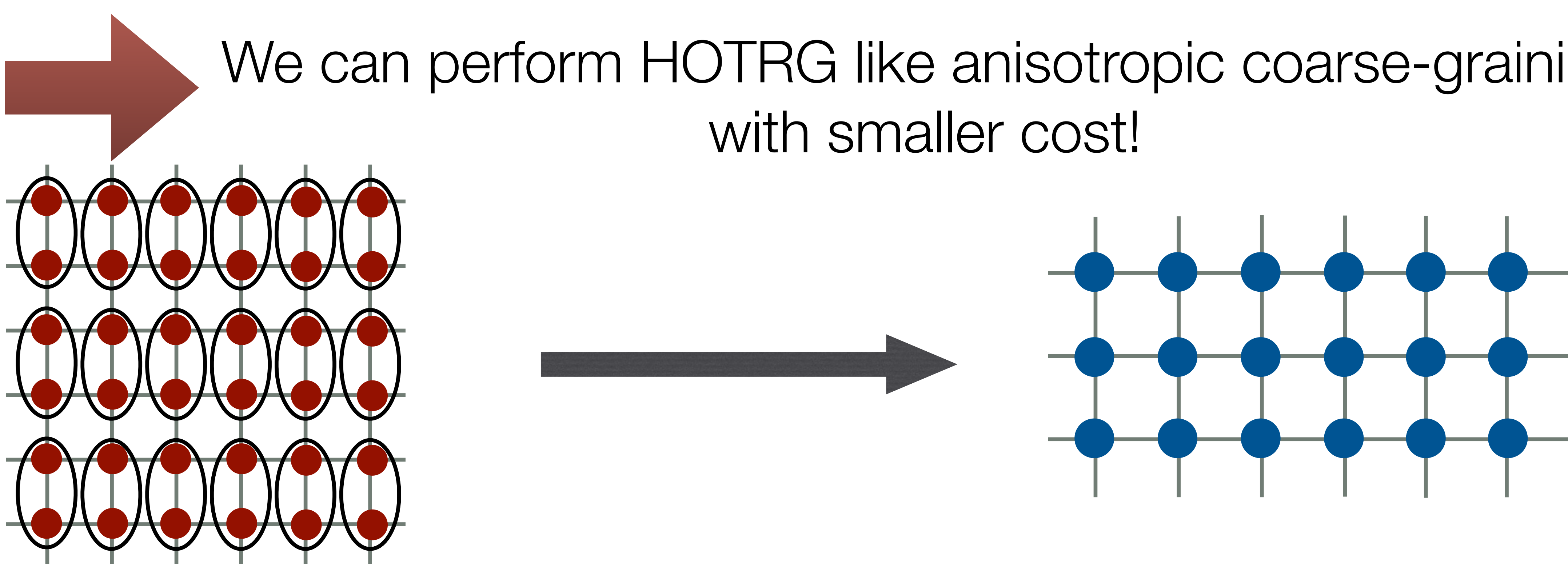
* We do not explicitly create 4-leg tensor.
(We need only 3-leg tensors!)



Computation time: $O(D^5)$

* By using *partial SVD* technique, such as the Arnoldi method, we can reduce SVD cost.

We can perform HOTRG like anisotropic coarse-graining with smaller cost!



Generalization to high dimension

D. Adachi, T. Okubo, and S. Todo, arXiv:1906.02007

We can easily generalize this
to high dimensions

Key points:

Before coarse graining,
we decompose a local tensor
into two tensors.

*We may consider
more decompositions.

Memory storage: $O(D^{d+1})$

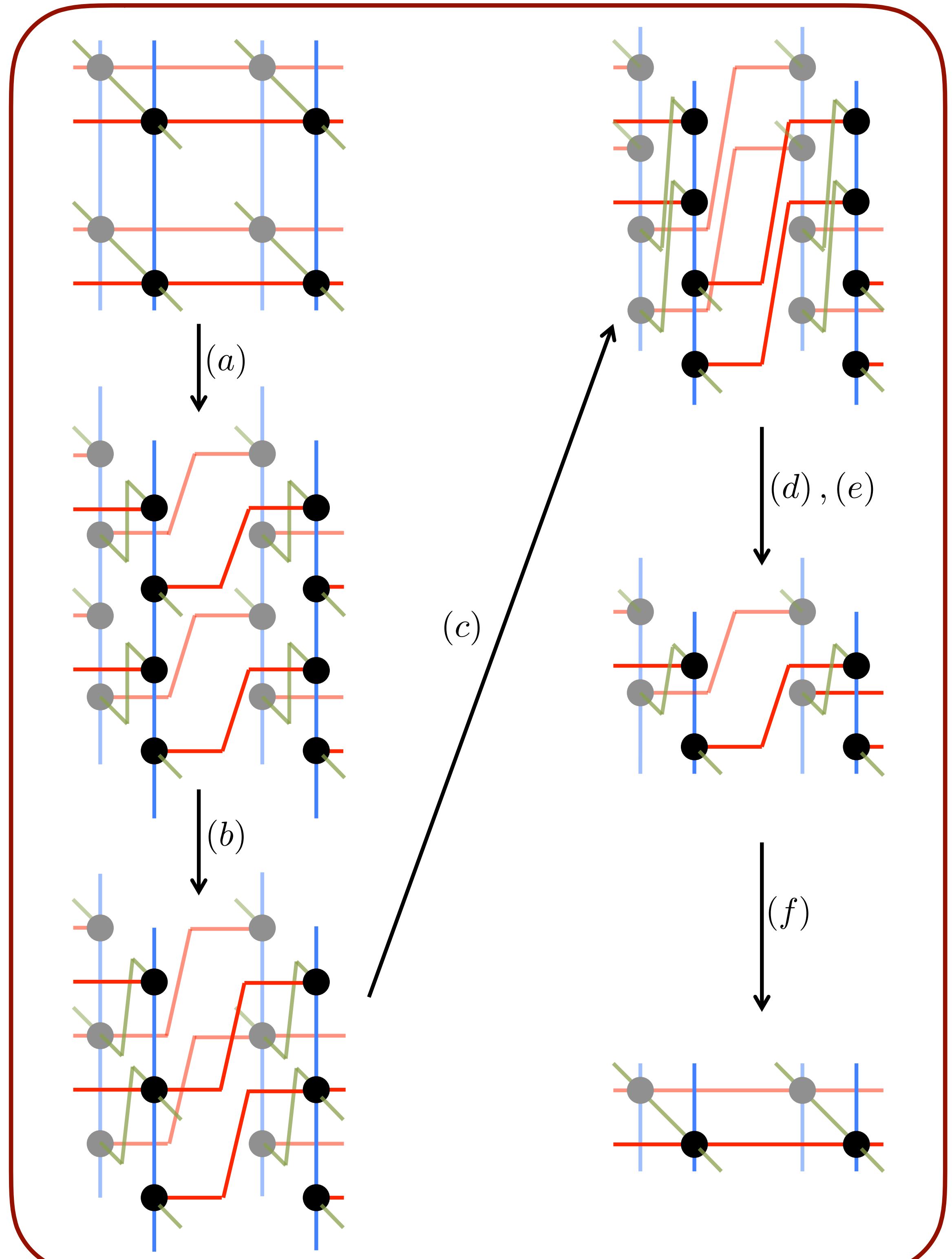
Computation time: $O(D^{2d+1})$

$\Uparrow$

HOTRG

Memory: $O(D^{2d})$

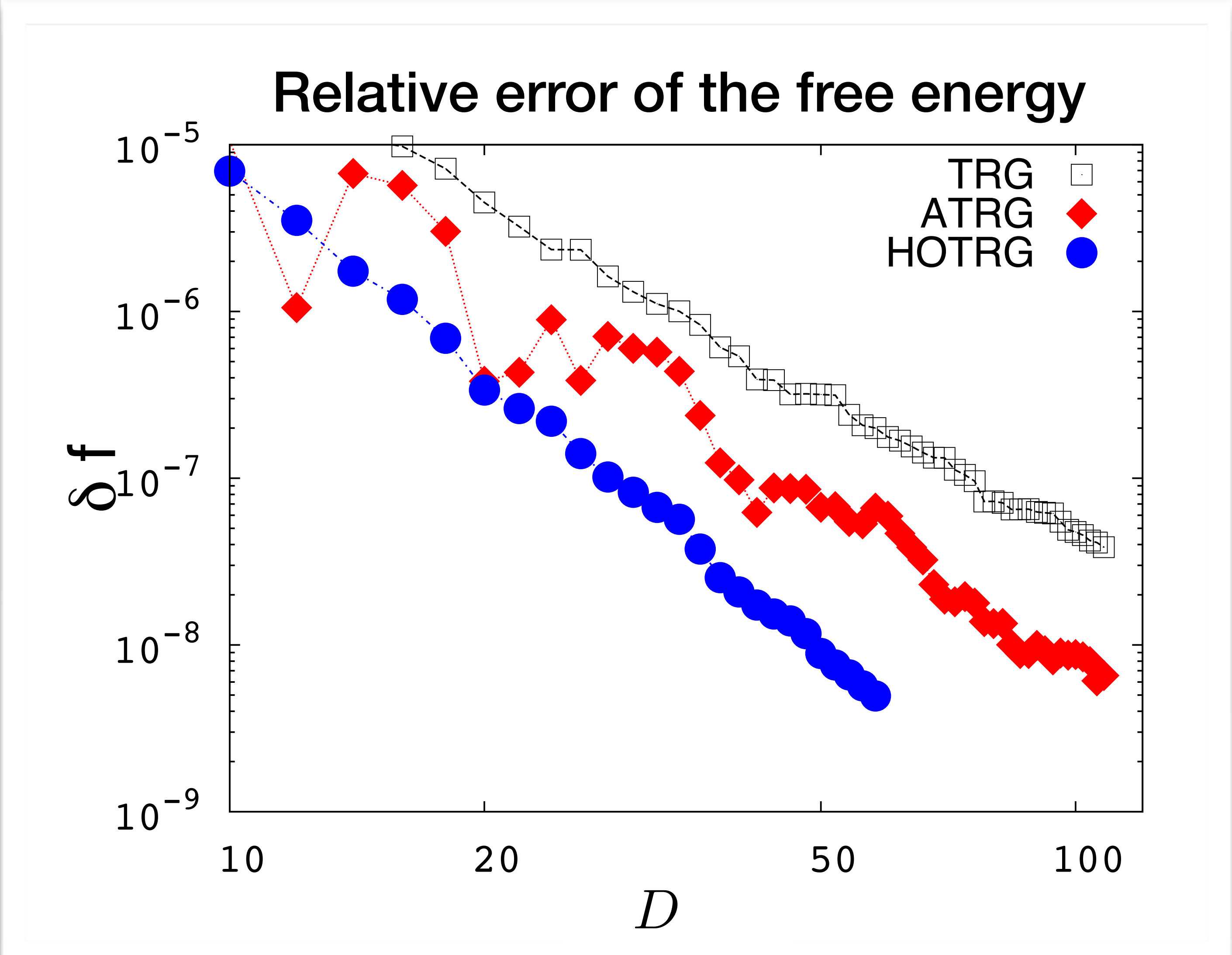
Time: $O(D^{4d-1})$



Benchmarks

D. Adachi, T. Okubo, and S. Todo, arXiv:1906.02007

2d square lattice Ising model at T_c



At the same bond dimension, accuracy of ATRG is between those of TRG and HOTRG.

HOTRG is better? → **No so straightforward!**

Benchmarks

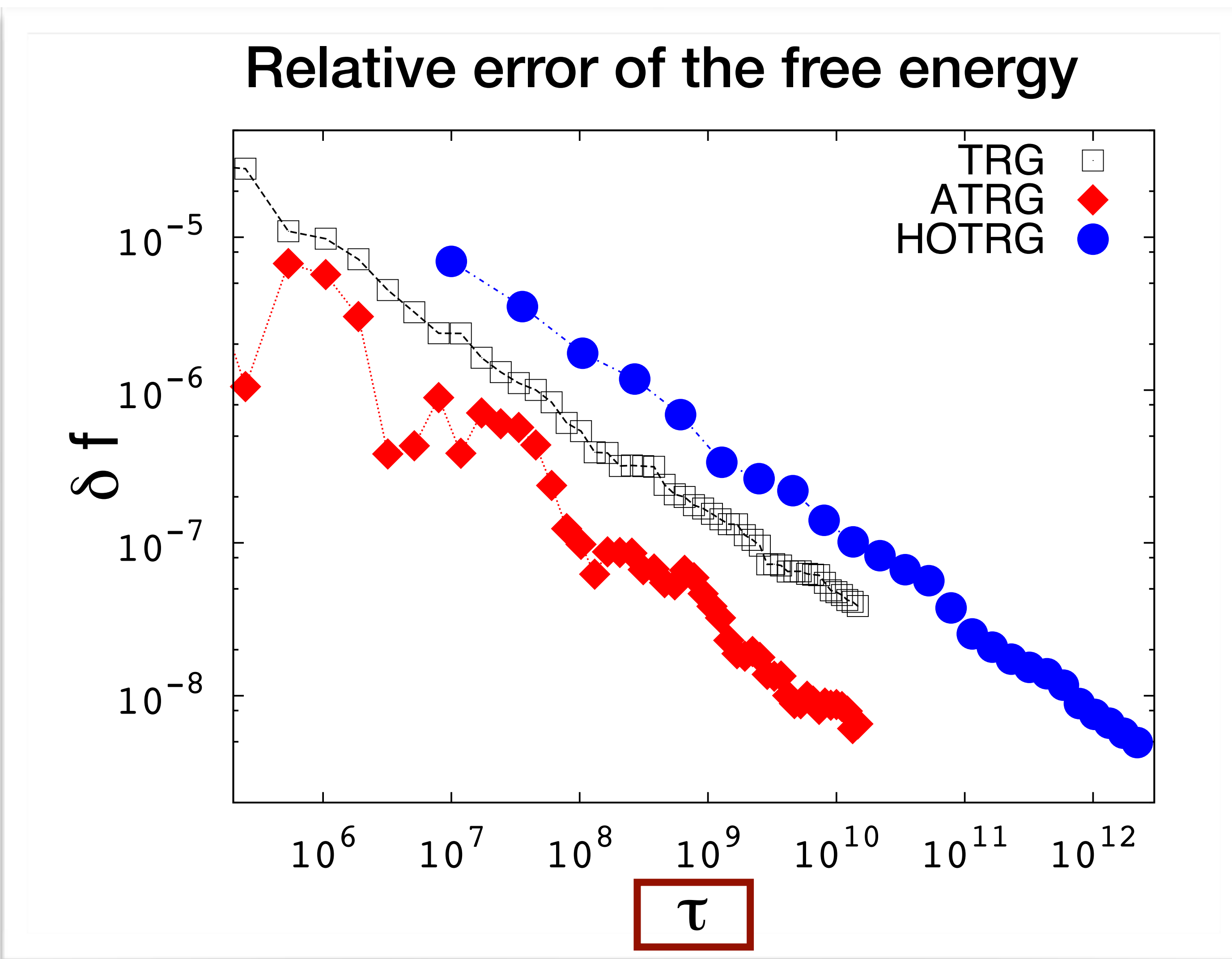
The computation costs are different between ATRG and HOTRG.

TRG, ATRG: $O(D^5)$

HOTRG: $O(D^7)$

Leading order computation time:

$$\tau \equiv \begin{cases} D^5 & \text{TRG and ATRG} \\ D^7 & \text{HOTRG} \end{cases}$$



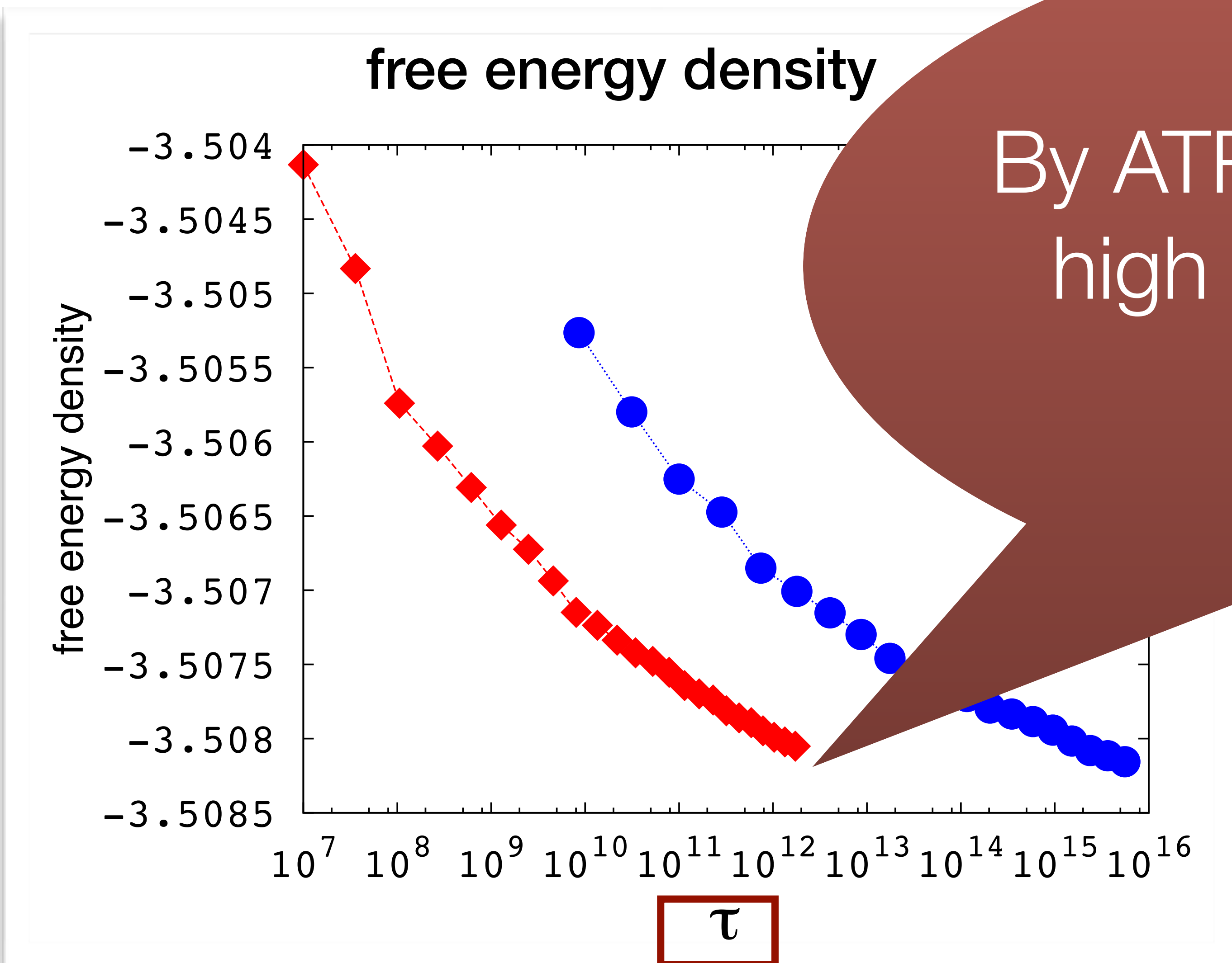
ATRG is the best!

(ATRG is ~10^2 faster than HOTRG!)

Benchmarks

D. Adachi, T. Okubo, and S. Todo, arXiv:1906.02007

3d square lattice Ising model at T_c



By ATRG, we can investigate high dimensional systems much easier!

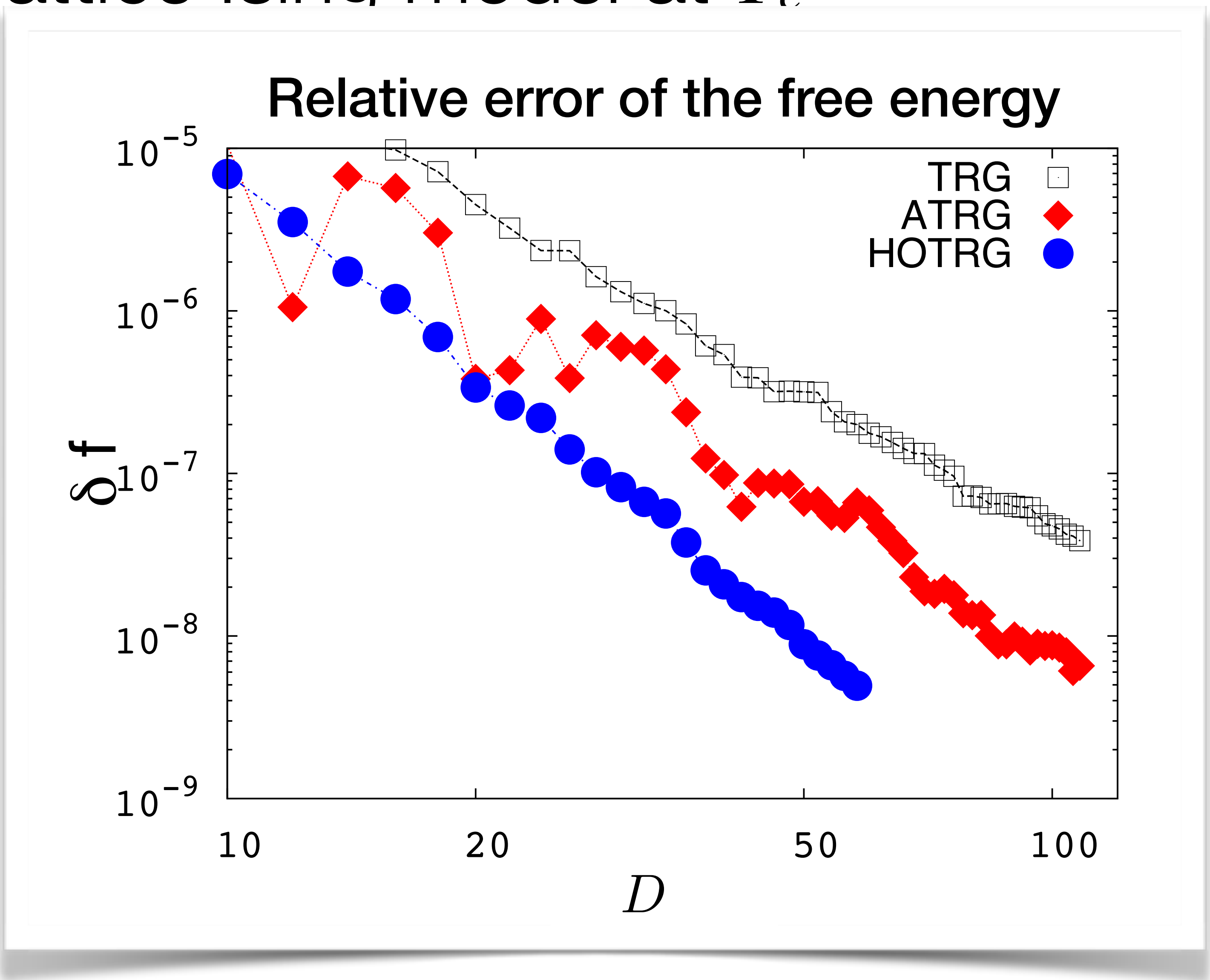
In order to obtain same free energy value, ATRG needs ~10^3 times smaller computation cost than HOTRG.

Bond-weighted TRG = BTRG

D. Adachi, T. Okubo, and S. Todo, in preparation

Why ATRG is better than TRG?

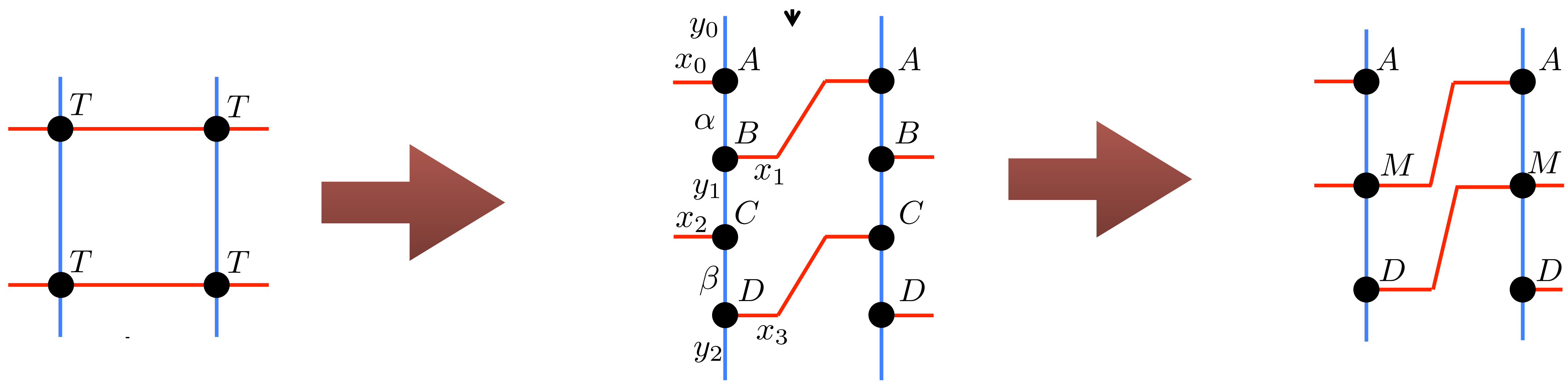
2d square lattice Ising model at T_c

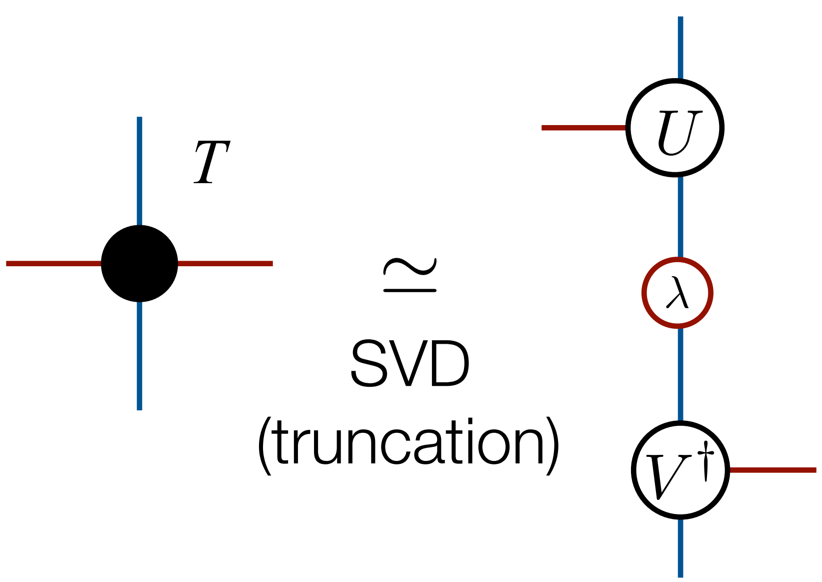


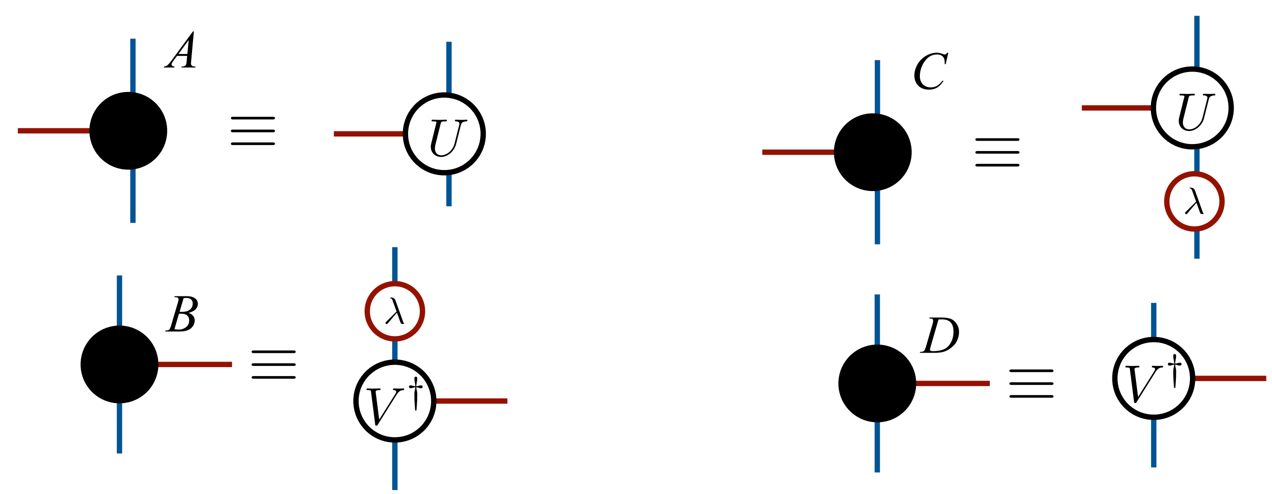
ATRG is better than TRG, although the cost is same.

Why?

Key point: weight of bonds



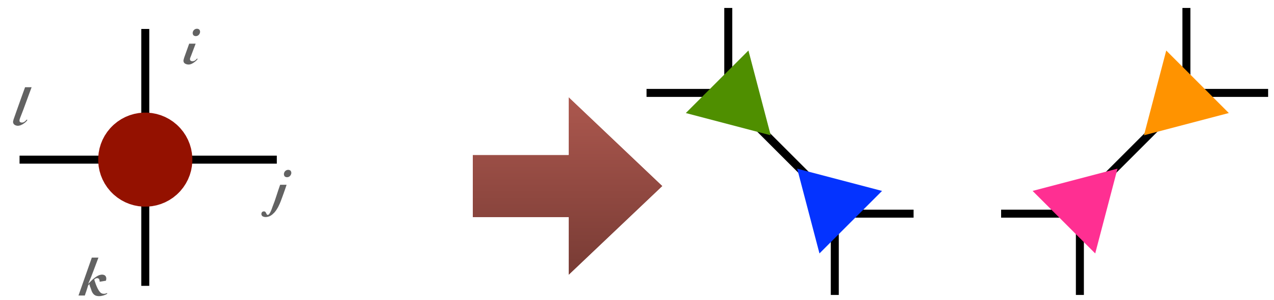




In ATRG, we assign singular values to only one tensor.

In TRG, usually it is divided to both tensors.

Disadvantage of local SVD truncation



$A_{i,j,k,l}$

Truncation by SVD

Local SVD is not necessarily the best "D-rank" approximation
for the whole TN (the partition functions).

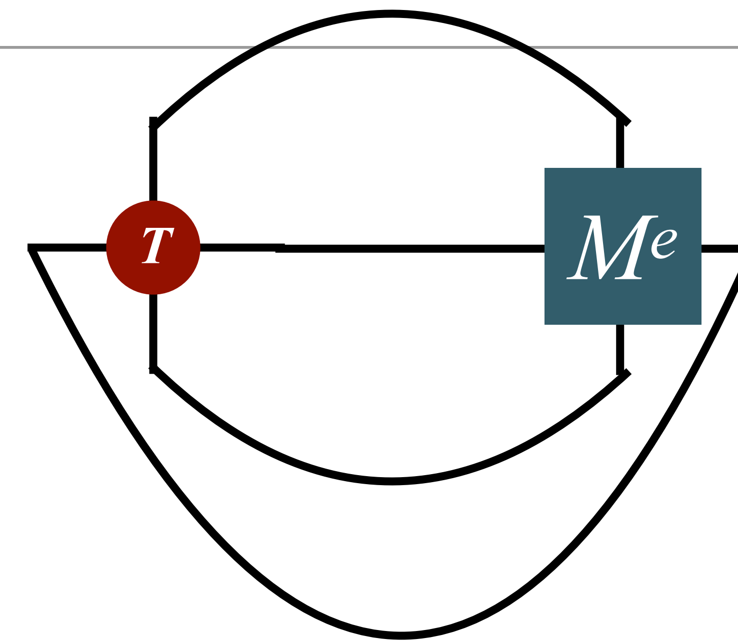
We need to consider the "environment"
for better truncations.

This is the key idea of **SRG (second renormalization)**

Z. Y. Xie, et al Phys. Rev. Lett. **103**, 160601 (2008).

Basic strategy of SRG

Z. Y. Xie, et al Phys. Rev. Lett. **103**, 160601 (2008).

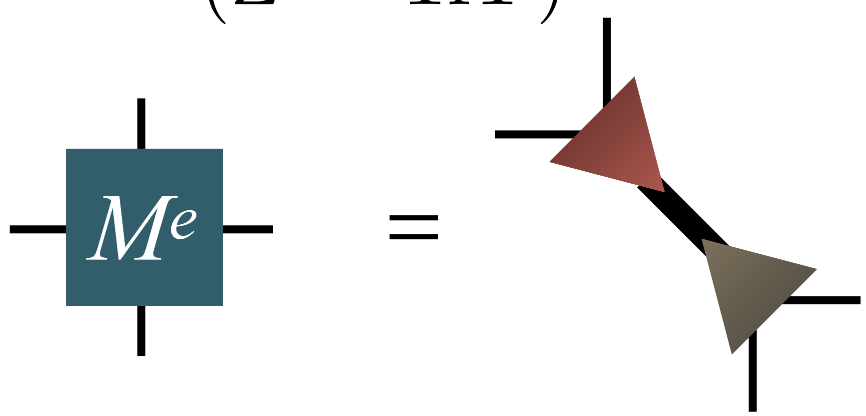


$$Z = \mathrm{Tr}TM^e$$

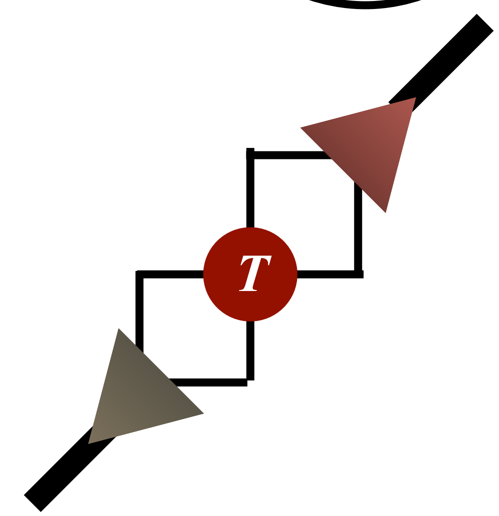
M^e: Environment tensor

$$\tilde{T} = (M^e)^{1/2}T(M^e)^{1/2}$$

$$(Z = \mathrm{Tr}\tilde{T})$$



$\tilde{T} =$



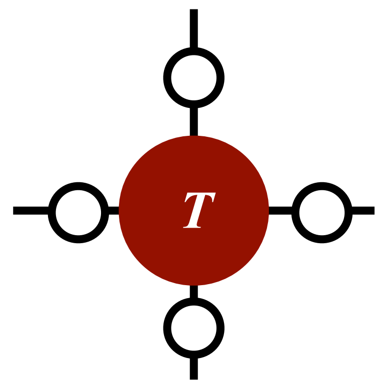
Better truncation: truncation based on SVD of $\tilde{T}$

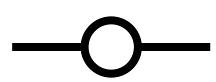
Disadvantage of SRG:

In order to obtain M^e we need iterative calculations,
which increases computation cost.

Mean field environment

As an approximated M^e, we might use "weight" of bonds as mean-field environment.

$\tilde{T} =$ 

 : Diagonal non-negative matrix

How can we estimate the bond weight?

- We can estimate it from the singular values of a local tensor.
  1. The weights are repeatedly improved by the singular values of $\tilde{T}$.

     Mean-field SRG. H.-H. Zhao et al, PRB **81**, 174411 (2010).
  2. Alternatively, we can consider the renormalization of extended tensor networks which contains bond weight explicitly.

→ **Bond-weighted TRG**

Summary

- We introduced Anisotropic TRG (**ATRG**)
  - By using ATRG, we can contract a d-dimensional TN with $O(D^{2d+1})$ computation cost and $O(D^{d+1})$ memory.
  - When we look *accuracy per computation time*, ATRG is much more efficient than HOTRG.
- We considered TRG of bond-weighed TN (**BTRG**)
  - By choosing proper exponent, BTRG shows better accuracy than HOTRG.
  - BTRG is easily generalized for other lattices and HOTRG.