TNSAA/Taipei: 2019/Dec. 猫空物理

Corner transfer matrix and Lattice Unruh effect for the XXZ chain

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introduction

Entanglement in quantum many body systems

- Quantum spin chain: bipartitoning entanglement
  entanglement spectrum/Hamiltonian

Characterizing entanglement between subsytems

DMRG/TNs c.f detector of SPT states

Our aims XXZ chain.

- analogy with Unruh effect
  the simplest gravitational effect on QFTs
- visualization of the entanglement
  classical world lines of spins (QMC)

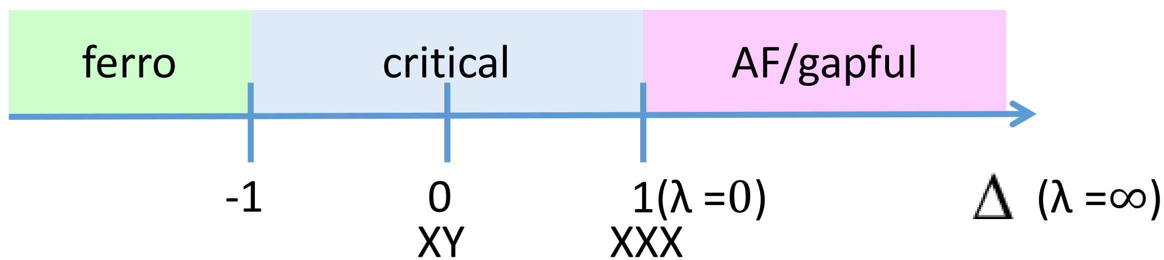
CTM for 6-vertex model/Lorenz boost operator integrability

Ising-like XXZ chain

$$\lambda > 0 \quad (\Delta > 1)$$

$$\mathcal{H} = J_\lambda \sum_{n=-L+1}^{L} \left[S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + \Delta S_n^z S_{n+1}^z \right]$$

$$J_\lambda = \frac{2}{\sinh \lambda} \qquad \Delta = \cosh \lambda$$



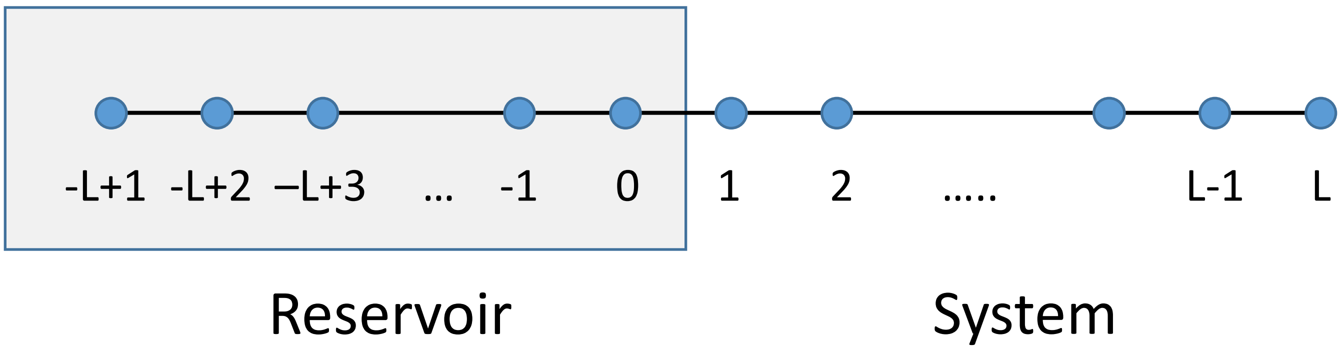
<u>Bethe ansatz solvable</u>

Bulk energy, excitation gap, magnetization, etc.

Ising-like regime:

The groundstate is gapful with a finite correlation length.

entanglement Hamiltonian for biparitioning



$$\rho = \sum_{n_-} \Psi^\dagger(n_-, n_+)\Psi(n_-, n_+) \quad \Rightarrow \quad S = -\mathrm{Tr}[\rho \log \rho]$$

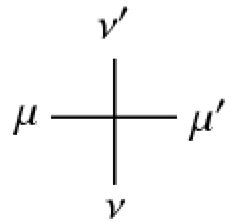
* This bipartition EE can be easily calculated by DMRG.

If we can write $\rho \sim \exp(-H_{\mathrm{EE}})$, H_{EE} is called entanglement Hamiltonian or modular Hamiltonian.

A modular Hamiltonian defines a time evolution in an angular direction different from the conventional time.

XXZ chain and 6-vertex model

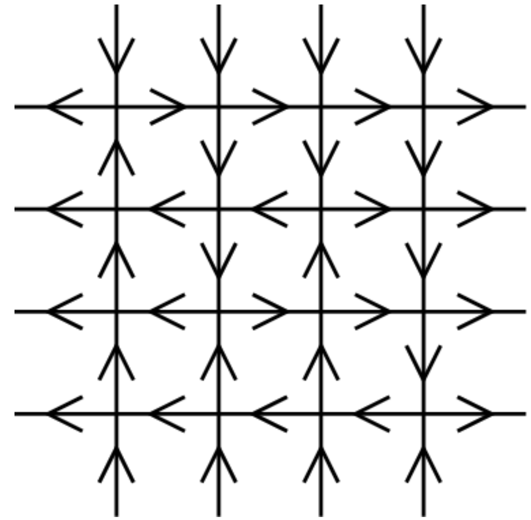
$$W(\mu,\nu|\mu',\nu') =$$



$$W(+,+|+,+) = W(-,-|-,-) = 1$$

$$W(+,-|-,+) = W(-,+|+,-) = \frac{\sinh(u)}{\sinh(\lambda-u)}$$

$$W(+,-|+,-) = W(-,+|-,+) = \frac{\sinh(\lambda)}{\sinh(\lambda-u)}$$



satisfies Yang-Baxter relation

$\lambda > 1$ Ising-like anisotropy = antiferroelectric regime

Commuting transfer matrices

$$[T(u), T(u')] = 0 \qquad T(u) = \sum_{\{\mu\}} \prod_n W_n(\mu_n, \nu_n|\mu_{n+1}, \nu_{n+1})$$

u : rapidity(=spectral parameter= pseudo momentum)

Hamiltonian of the XXZ chain $\quad \mathcal{H} = -\frac{d}{du}\log T(u)\Big|_{u=0}$

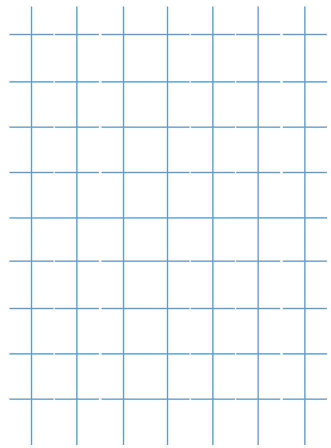
Simultaneous eigenstate $\quad [T(u), \mathcal{H}] = 0$

integrability and CTM

<u>Eigenvector: ~~Bethe type~~</u>

Baxter's magic / CTM

$$|\Psi\rangle \sim \lim_{n\to\infty} T^n |b\rangle$$ =



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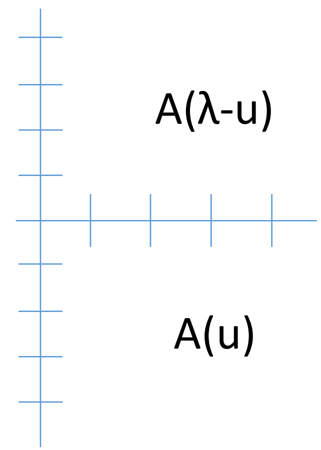


Baxter, J.Math.Phys. (1968), J.Stat.Phys. (1971)

The groundstate wavefunction can be written as a product of CTMs

$$\Psi \sim A(\lambda - u)A(u) \quad \text{with} \quad A(u) \sim e^{-u\mathcal{K}}$$

<u>Reduced density matrix</u>

$\mathcal{K}$ plays a role of the entanglement Hamiltonian

$$\rho = \exp(-\beta_\lambda \mathcal{K})/Z$$

with

$$\beta_\lambda \equiv 2\lambda$$

$$Z \equiv \mathrm{Tr}\exp(-\beta_\lambda \mathcal{K})$$

entanglement/corner Hamiltonian

$$\mathcal{K} \equiv J_\lambda \sum_{n=1}^{L} n \left\{ S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + \Delta S_n^z S_{n+1}^z \right\}$$

Free boundary condition at n=1, L

The boundary effect at n=1 should be perfectly suppressed at $\beta_\lambda \equiv 2\lambda$ to reproduce the uniform ground state.

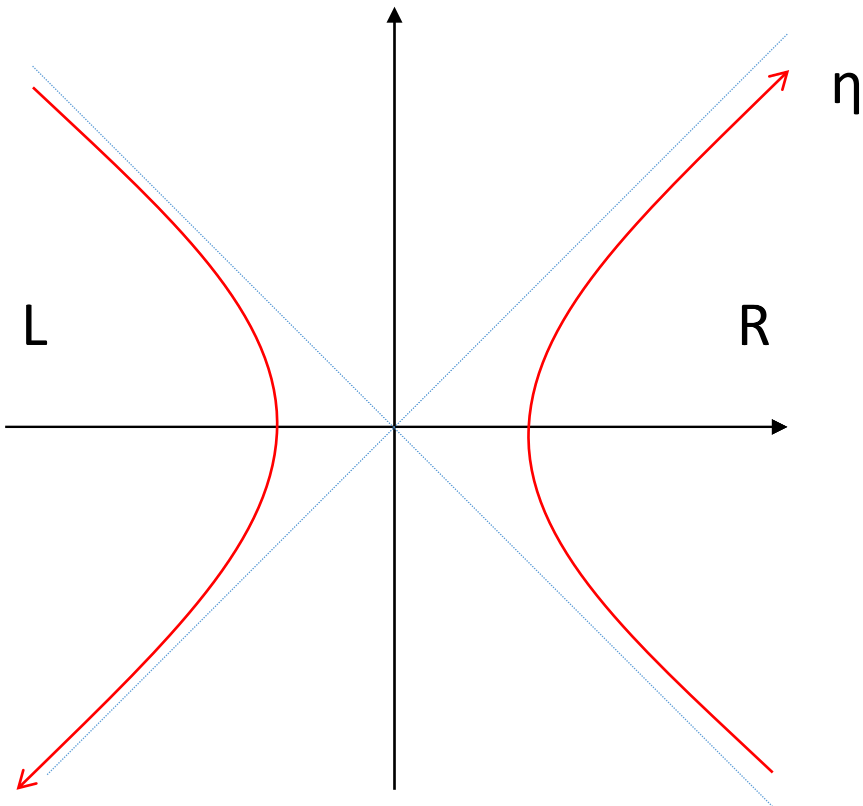
The energy scale is proportional to n

Effective temperature decreases as n increases.
(This can be a source of difficulty in a QMC simulation)

The exact spectrum of corner Hamiltonian provides the exact EE for XXZ chain. Kaulke Peschel, K.O. Y. Hieida and Y. Akutsu

What is the interpretation of the corner Hamiltonian?

Unruh effect : vacuum fluctuation in quantum field theory



The Left and right parts are space-like regimes, which are classically separable!

A constantly accelerating observer

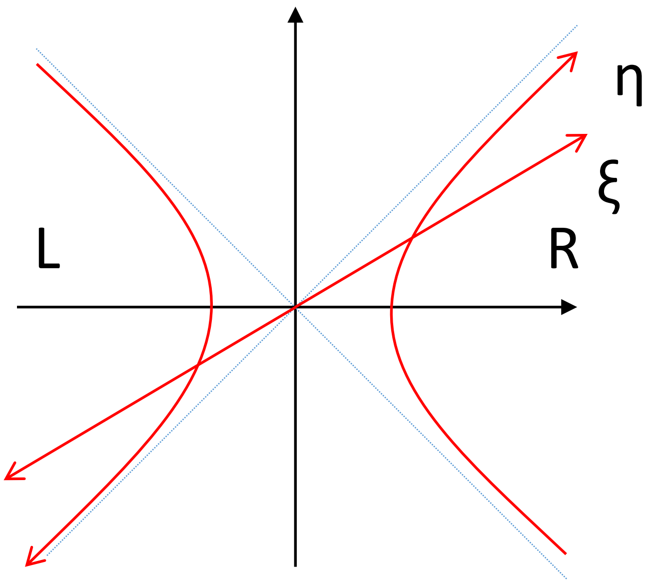
$$
\begin{aligned}
x &= \frac{e^{a\xi}}{a}\cosh(a\eta) \\
t &= \frac{e^{a\xi}}{a}\sinh(a\eta)
\end{aligned}
$$

sees the vacuum as a thermalized state with an effective temp. (Unruh temp.)

$$\beta^* = \frac{2\pi}{a}$$

The quantum fluctuation of the vacuum is observed as thermal fluctuation

Rindler-Fulling quantization (η,ξ)



constantly accelerating observer

R
$$x = \frac{e^{a\xi}}{a}\cosh(a\eta)$$
$$t = \frac{e^{a\xi}}{a}\sinh(a\eta)$$

$$K = \int d\xi\, \xi \frac{1}{2}[\frac{(\partial_\eta \phi)^2}{\xi^2} + (\partial_\xi \phi)^2 + m^2\phi^2]$$

Lorentz boost operator

$$b_p^{\mathrm{R}}|0\rangle_R = b_p^{\mathrm{L}}|0\rangle_L = 0$$

$$H = \int dx\, \frac{1}{2}[(\partial_\mu \phi)^2 + m^2\phi^2]$$

with $a_k|0\rangle_M$ Minkowski vacuum

c.f. $K = \int dx\, x\frac{1}{2}[(\partial_\mu \phi)^2 + m^2\phi^2]$

<u>Bogoliubov transformation</u>

$$|0\rangle_M = e^{-\prod_p e^{-\pi p/a} b^{L\dagger}_p b^{R\dagger}_p} |0\rangle_L |0\rangle_R$$

$$\rho_R = \mathrm{Tr}_L |0\rangle_{MM}\langle 0| = \prod_p e^{-\beta^* K_p}$$

with $\beta^* = \dfrac{2\pi}{a}$ and $K_p = p b_p^{\mathrm{R}\dagger} b_p^{\mathrm{R}}$

m=0 massless case

$\mathcal{K}$ corner Hamiltonian /Hamiltonian of CTM

$$\mathcal{K} \equiv J_\lambda \sum_{n=1}^{L} n \left\{ S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + \Delta S_n^z S_{n+1}^z \right\}$$

Lattice Lorentz boost operator
(Rapidity shift operator)

$$A(-\mu)T(\nu)A(\mu) = T(\mu + \nu)$$

$\Rightarrow$ The CTM formulation corresponds to the Rindler quantization of the relativistic quantum field theory

Lattice Poincare algebra

H.B.Thacker, Physica D **18**, 348 (1986).

$$[P, \mathcal{H}] = 0\,, \quad [\mathcal{K}, P] = iH\,, \quad [\mathcal{K}, H] = i\tilde{I}_2 \ \sim O(a^2)$$

$$I_0 = iP \quad I_1 = -\mathcal{H} \quad \tilde{I}_2 = iI_2 = \sum [h_{n,n+1}, h_{n+1,n+2}] \quad \log T(u) = \sum \frac{I_n}{n!} u^n,$$

<u>Reduced density matrix</u>

$\mathcal{K}$ plays a role of the entanglement Hamiltonian

$$\rho = \exp(-\beta_\lambda \mathcal{K})/Z$$

with

$$\beta_\lambda \equiv 2\lambda$$

$$Z \equiv \mathrm{Tr}\exp(-\beta_\lambda \mathcal{K})$$

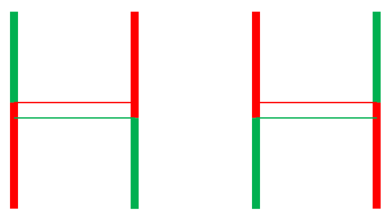
Unruh effect	Rindler-Fulling quantization (free scalar field)	
entanglement Hamiltonian	Lorentz boost operator	proper time evolution = momentum shift
parameter	acceleration a mass m independent	

XXZ chain	CTM/RDM diagonal basis	
entanglement Hamiltonian	Corner Hamiltonian = lattice Lorentz boost	Angular time evolution = rapidity shift
parameter	anisotropy λ both of the effective acceleration and mass gap are defined by λ	

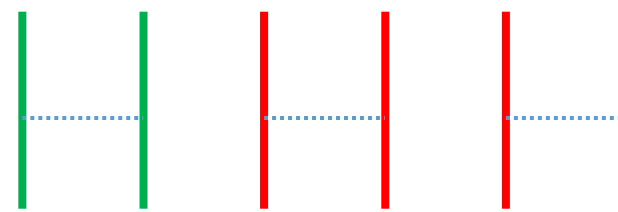
extracting entanglement from the corner Hamiltonian

Finite temperature, no furstration

⇨ WL-QMC



off-diagonal interaction
(XY-terms)



diagonal interaction
(zz terms)

Stochastic updating of the classical world-lines provides typical configuration of spins in the equillibrium.

world-line entanglement

- The corner Hamiltonian defines the imaginary angular time evolution

  time evolution at n $\quad \Delta\tau \propto n$
- Scaling of the imaginary time into the angular time defines an effective acceleration for the XXZ chain.

Scale imaginary time: τ $\quad \theta = a\tau \quad$ with $\quad a = \dfrac{2\pi}{\beta_\lambda} = \dfrac{\pi}{\lambda}$

$$0 \le \tau < \beta_\lambda \quad \Rightarrow \quad 0 \le \theta < 2\pi$$

a : effective acceleration

θ : angular time

a=0 : classical limit

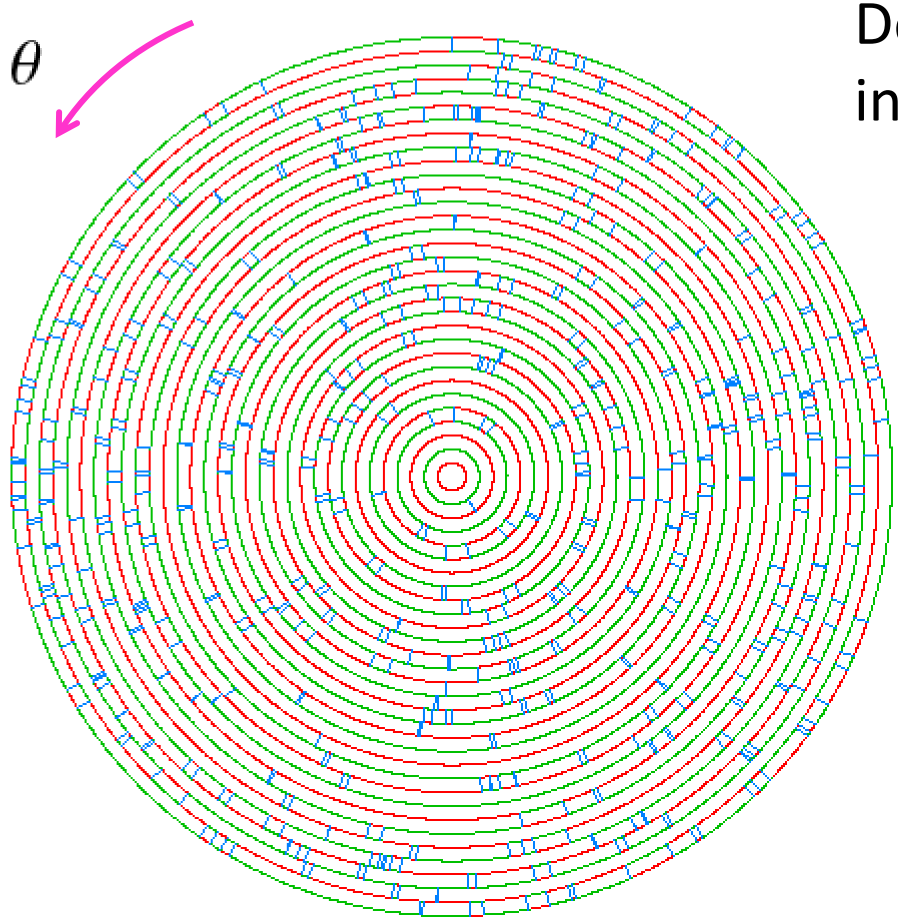
- We can illustrate entanglement as circles of classical world-lines surrounding the entangle point(n=0)

snapshots

$\Delta = 2.0$ $\beta_\lambda \equiv 2\lambda$

$(\lambda = 1.3169\cdots)$

How can the "uniform" ground state be realized for the non uniform Hamiltonian?



Density of kinks looks uniform in this plot!

Local temp $\propto n\beta_\lambda$

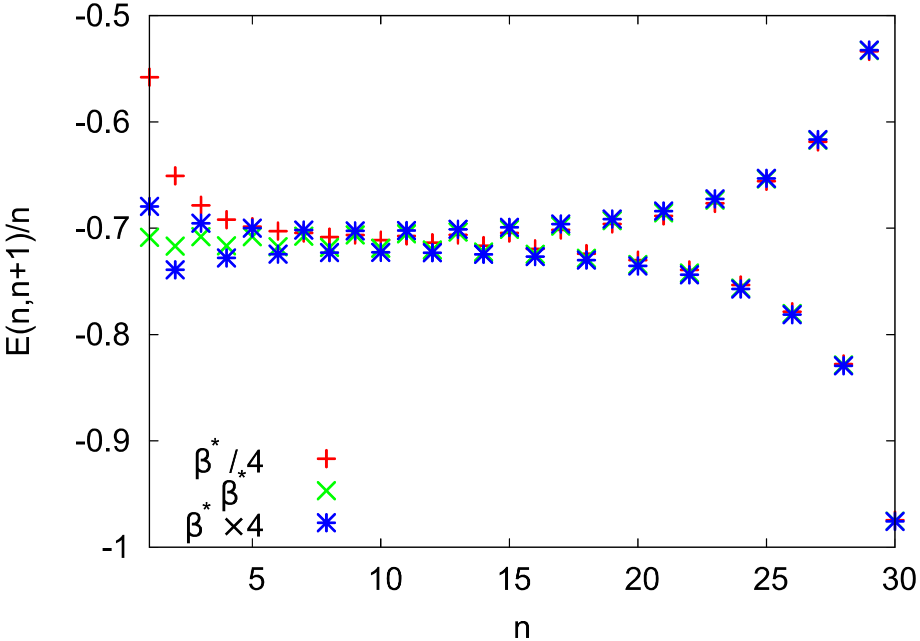
kink # $\propto n\beta_\lambda$

For $\beta < \beta_\lambda$ (high temp.), kinks around the center becomes space.

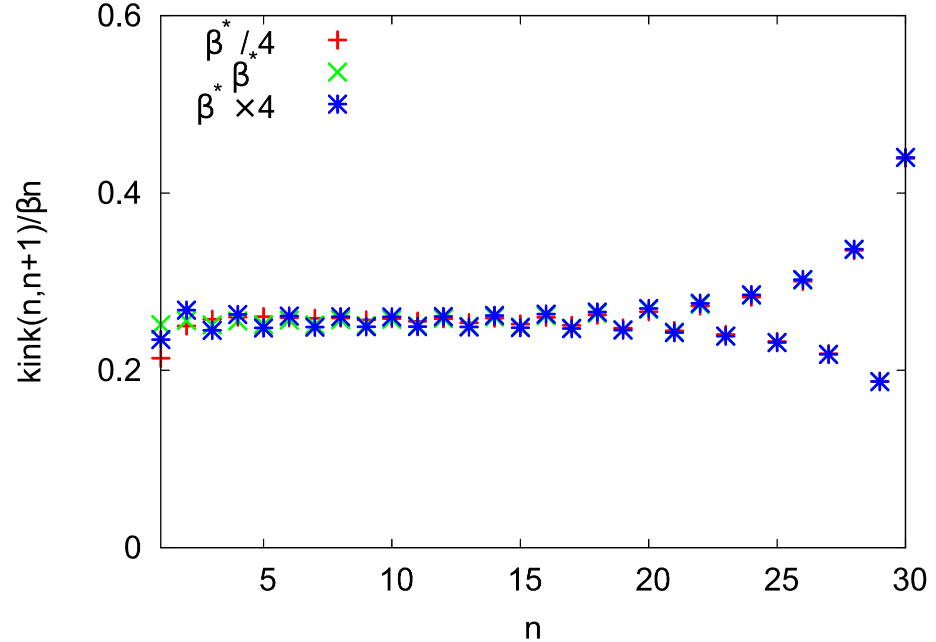
For $\beta > \beta_\lambda$(low temp.), kinks around the center are oscillating.

bond energy distribution $\Delta = 2.0$

$\langle (S_n \cdot S_{n+1})_\Delta \rangle$ normalized bond energy



normalized kink density

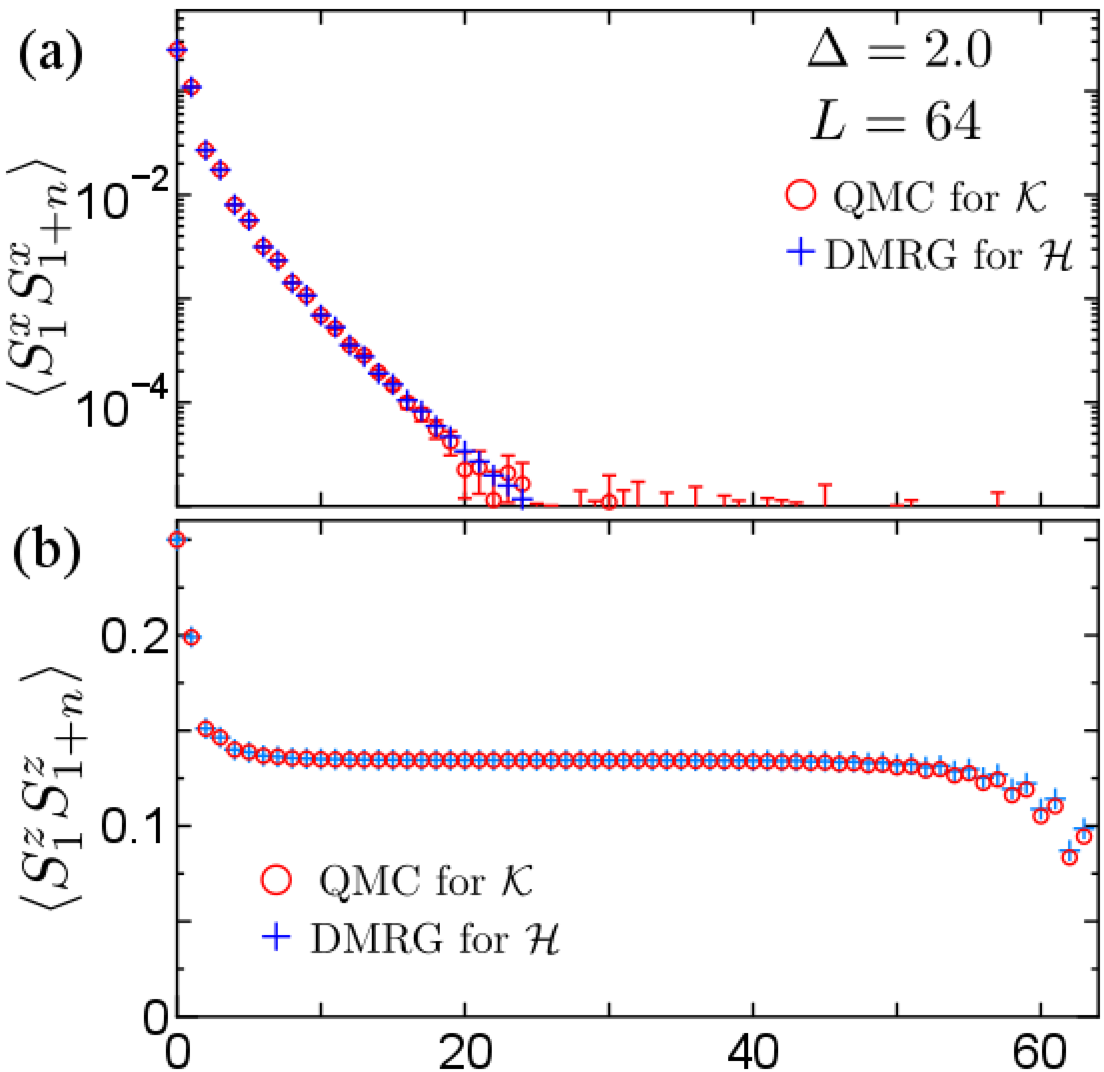


kink density is related to the off-diagonal parts of local energy

At $\beta = \beta_\lambda$, the normalized bond energy and kink density become flat around n=1

reproducing uniform ground state wavefunction.

correlation functions $\Delta = 2.0$



Perfect correspondence to the DMRG results for the groundstate of H

Entanglement Entropy

The groundstate entanglement entropy for H can be calculated as the thermal entropy for the entanglement Hamiltonian.

$$S_{\mathrm{EE}} = -\mathrm{Tr}_S[\rho \log \rho] = \beta_\lambda \langle \mathcal{K} \rangle + \log Z$$

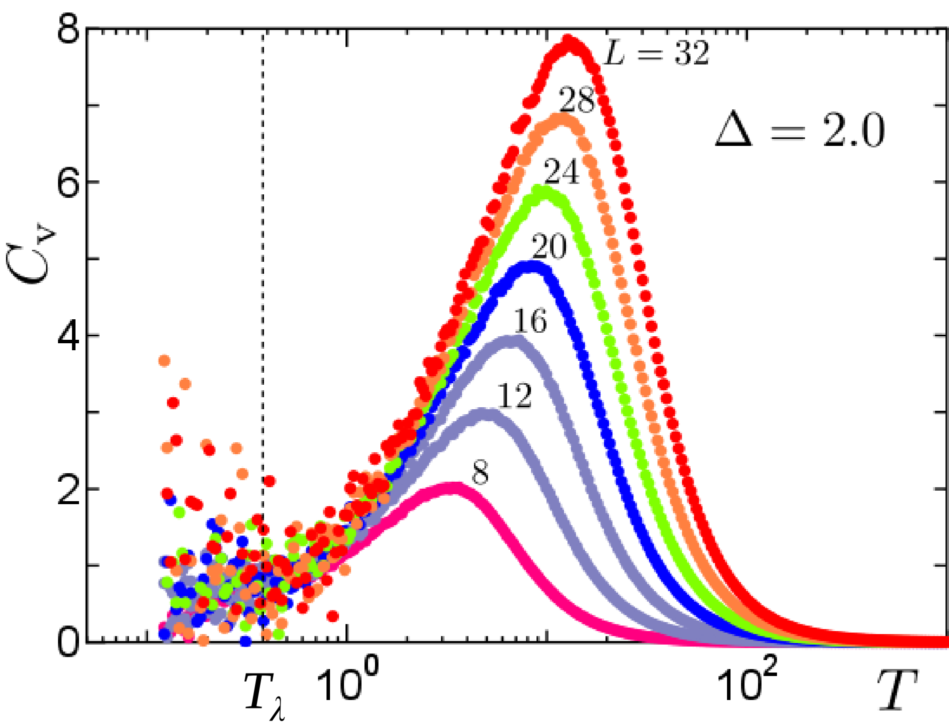
We calculate S_EE with integration of a specific heat estimated by a QMC simulation.

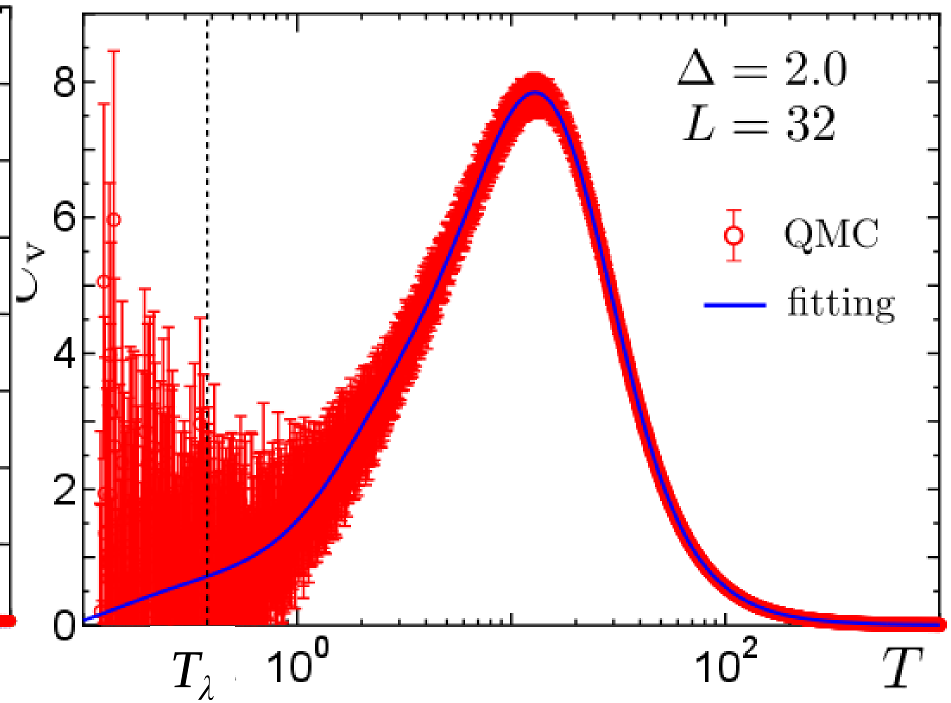
$$S_{\mathrm{EE}} \;= L \log 2 - \int_{T_\lambda}^{\infty} \frac{C_{\mathrm{v}}}{T} dT = L \log 2 - \int_{\log T_\lambda}^{\infty} C_{\mathrm{v}} dx$$

The estimation of the entropy is not easy but possible with QMC.

Cv

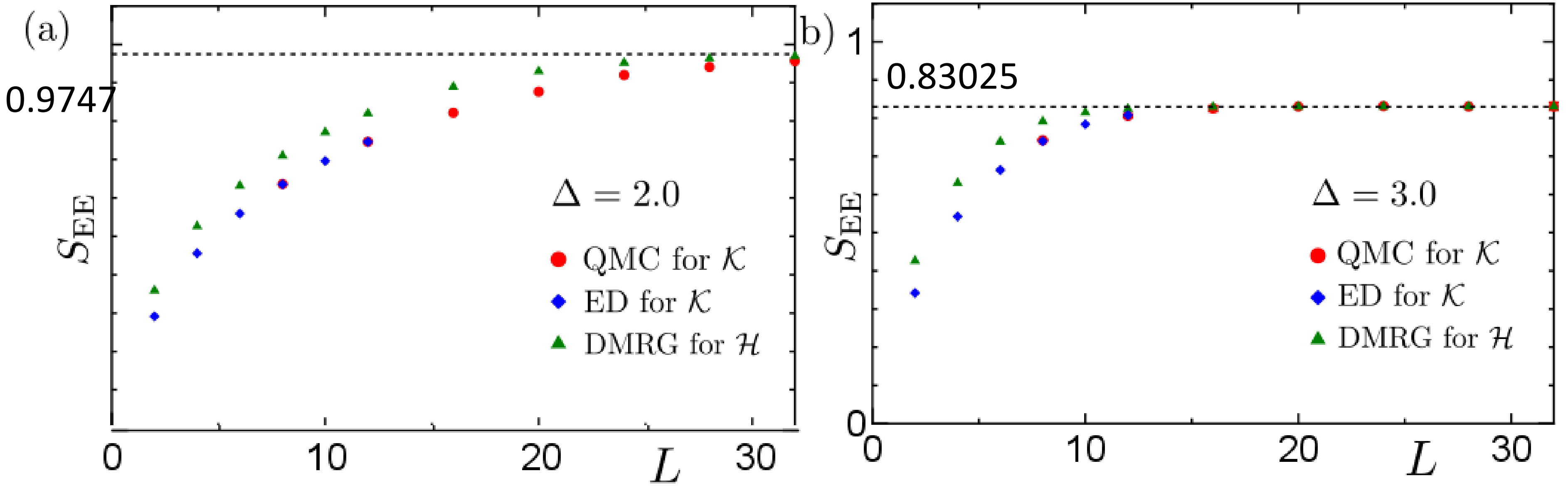
Fitting: Gaussian Kernel method





Eneanglement Entropy

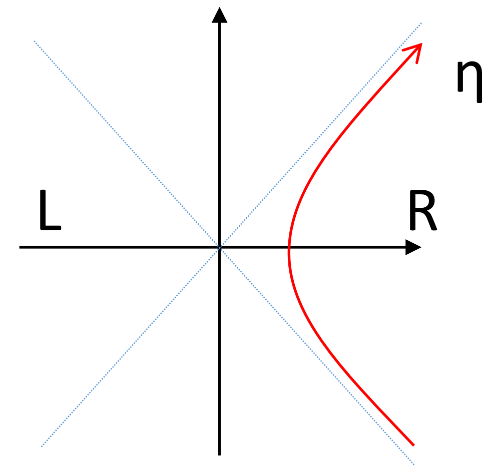
$\Delta = 2.0$ $\Delta = 3.0$



Estimation of EE approaches to the exact value of EE for the half0infinite subsystem

The deviation from the DMRG result originates from geometry of world sheets:
DMRG: cylinder, corner Hamiltonian: disk

Unruh-DeWitt detector



A harmonic oscillator coupled with a scalar field moving along the Rindler trajectory

$$S = \int d\eta \phi(x(\eta), t(\eta)) \hat{X}(\eta) \qquad x = r\cosh(a\eta)\,, t = r\sinh(a\eta)$$

➡ This detector is excited by the thermalized vacuum.

Excitation rate is given by an integration of the Wightman function

$$\Rightarrow \quad P_n \propto \int d\eta e^{i\omega_n \eta}\, {}_M\langle \phi(x(\eta), t(\eta)) \phi(r, 0) \rangle_M$$

Capturing the Bose distribution with the Unruh temp.

$$P_n \propto \frac{1}{e^{\beta_U \omega_n} - 1}$$

(massless case)

XXZ-chain analogue of the detector

A harmonic oscillator coupled with a spin in the XXZ chain?

But, the detector does not accelerate in the chain literally .

Scalar field $$\phi(x(\eta), t(\eta)) = e^{ia\eta L}\phi(r, 0)e^{-ia\eta L}$$

η-dependent Lorentz transformation

Spin coupled with the detector : $\mathcal{K}$ lattice Lorentz boost

$$S_n^{\mu}(\eta) = e^{-ia\eta\mathcal{K}} S_n^{\mu} e^{ia\eta\mathcal{K}}$$

$n \sim r$: distance from the entangle point

Autocorrelation function with respect to η

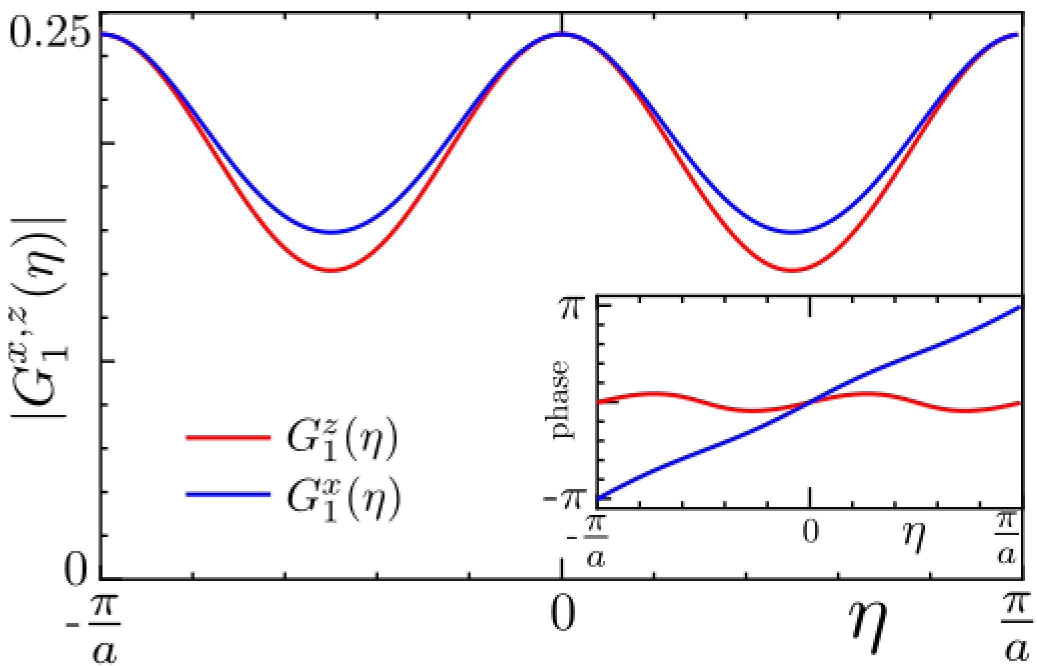
$$G_n^{\mu}(\eta) \equiv \frac{\mathrm{Tr}\, S_n^{\mu}(\eta) S_n^{\mu}(0) e^{-\beta_\lambda \mathcal{K}}}{Z}$$

Autocorrelations

DMRG: Renormalization transformation matrix gives
the relation between the $\mathcal{K}$ diagonal bases and the usual spin bases

Bogoliubov trans. (Rindler) (Minkowski)

n=1



classical value

π/a periodicity

Imaginary shift
of the rapidity
+
lattice effect

summary

arXiv:1906.10441

J. Phys. Soc. Jpn. **88**, 114002 (2019)

- We calculate the groundstate properties of the Ising-like XXZ chain with a finite temperature formulation based on the entanglement Hamiltonian/CTM.

Lattice Unruh effect

- We can understand the entanglement from the viewpoint of classical world lines surrounding the entangle point

world-line entanglement

- Can we realize lattice Unruh-Dewitt detector?

Autocorrelation captures entanglement spectrum

entanglement detector

- Critical cases? CFT, SSD, numerically bad convergence